Planar auxeticity from various inclusions

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Motivation

What is an **auxetic**?

\( \nu < 0 \) (Negative Poisson’s Ratio) is for mechanics what negative refractive index is for optics \( n < 0 \).

Auxetics are metamaterials.

Beneficial features from NPR

Resistance to shape change and indentation; crack resistance; better vibration absorption (including acoustic one); synclastic curvature, different dynamics.

Applications of NPR materials

Medicine (stents, bandages, implants), defence (energy absorption), furniture industry (better mattresses \( \rightarrow \) indentation), automotive industry and sports (safety belts).
Poisson’s ratio (stretching along $x$ axis)

Formal definition

$$\nu_{xy} = -\frac{\varepsilon_{yy}}{\varepsilon_{xx}} = \frac{s_{yyxx}}{s_{xxxx}}$$

Engineering definition

$$\nu = -\frac{\Delta H}{\frac{H_0}{\Delta W} W_0}$$
Motivation – rotating units

Motivation – rotating units


Our result from 2010
Motivation – rotating units


Our result from 2010

Motivation – rotating units


Our result from 2010


Why not fill the gaps?
Structures

Geometry:
- Periodically arranged ellipses
- Alternating axes (perpendicular)

Repeating unit cell (RUC):
- 4 rotating units
- Periodic mesh

Boundary conditions within **reduced RUC**
- All boundaries remain straight lines
- Less computational resources needed
Abaqus/CAE via Python interface generates inp files (wc -l ellipses-uniform-mesh.py gives 640 lines),
Abaqus/STANDARD performs simulations generating odb files,
Abaqus/CAE extracts important information from each odb file.

One run is enough to calculate $\nu$ as well as $E$ in one direction:

$$\nu_{\text{eff}} = -\frac{V_y/H_0}{U_x/W_0}. \quad (1)$$

$$E_{\text{eff}} = 2 \frac{E_{\text{elas}} W_0}{T \ U_x^2 H_0}. \quad (2)$$

- $V_y$ the displacement of (an arbitrary) top node
- $E_{\text{elas}}$ elastic deformation energy
- $T$ denotes the plane-stress thickness.
- $U_x$ known deformation
Meshes: uniform vs. non-uniform

The discretization (meshing) is an essential and most challenging step on FE analysis.
Meshes: uniform vs. non-uniform

The discretization (meshing) is an essential and most challenging step on FE analysis.
Mesh & convergence

Figure: Nonuniform exemplary mesh. Each edge was assigned a bias seed in order to obtain possibly dense mesh in the narrowest (bottleneck) regions. In this case $L_{\text{gap}}/d_{\text{min}} = 8$ was chosen in order to make the figure readable. In the simulations this ratio was 100. Fig. on the right shows that the convergence tests were performed up to the value $d_{\text{min}} = 10^{-4}$.

Figure: Convergence of Poisson’s ratio as a function of $d_{\text{min}}$ at fixed $d_{\text{max}} = 0.01$. Here $\nu_{\text{matr}} = \nu_{\text{incl}} = -0.99$, $E_{\text{matr}} = 1$. The geometric parameters are: $a = 0.09$, $b = 0.89$. The convergence displays various characteristics depending on the value of $E_{\text{incl}}$. 
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**Figure:** Convergence of $\nu_{\text{eff}}$ and $E_{\text{eff}}/E_{\text{matr}}$ as obtained within **non-uniform meshing** for $a = 0.09$, $b = 0.89$, $\nu_{\text{incl}} = \nu_{\text{matr}} = -0.99$ and **increasing mesh density**. The upper, monotonic curves refer to the right (Young’s modulus, red) axis, while the non-monotonic curves refer to the left (Poisson’s ratio, blue) axis. The numbers in the legend stand for the $L_{\text{gap}}/d_{\text{min}}$ ratio.
Detailed convergence – $\nu$

![Graph showing detailed convergence for $\nu$ with various inclusions and mesh densities. The graph depicts the effective Poisson's ratio $\nu_{\text{eff}}$ against the inverse of the mesh density $1/n_1$. The y-axis represents $\nu_{\text{eff}}$ values, ranging from $-0.45$ to $-0.7$, and the x-axis shows the mesh density inverses, ranging from $10^{-1}$ to $50^{-1}$. The data points indicate a trend with decreasing $\nu_{\text{eff}}$ as the mesh density increases.]
Detailed convergence – $\nu$

$\nu - 0.7 - 0.65 - 0.6 - 0.55 - 0.5 - 0.45$

$E_{\text{incl}} = 10^{-4}$

non-uniform mesh

uniform mesh

$100^{-1} 200^{-1} 50^{-1}$

$1/n_1$

$\eta_{\text{eff}}$

$d_{\text{max}} = 0.02$

$d_{\text{min}}$

$PR$

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Detailed convergence – ν

\[ \nu \approx 0.7 - 0.65 - 0.6 - 0.55 - 0.5 - 0.45 \]

\[ \frac{1}{n_1} = 10^{-1} \]

\[ 1000^{-1}, 500^{-1}, 200^{-1}, 100^{-1} \]

\[ E_{\text{incl}} = 10^{-4} \]

non-uniform mesh

uniform mesh

\[ d_{\text{eff}} = \frac{d_{\text{max}}}{n_1} \]

\[ PR \]

\[ d_{\text{max}} = 0.02 \]

\[ d_{\text{max}} = 0.01 \]

\[ d_{\text{max}} = 0.005 \]

\[ d_{\text{max}} = 0.0025 \]
Detailed convergence – $E$

$E_{\text{eff}}/E_{\text{matr}} = 10^{-4}$

$E_{\text{incl}} = 10^{-4}$

non-uniform mesh

uniform mesh
Required time vs. number of FE layers

![Graph showing required time vs. number of FE layers for different conditions, including UNIFORM and N-UNIFORM with different values of \(d_{\text{max}}\).]
Memory consumed vs. number of FE layers

![Graph showing memory consumption vs. \( L_{\text{gap}}/d_{\text{min}} \) or \( L_{\text{gap}}/d_u \)]

- **UNIFORM**
- N-UNIFORM, \( d_{\text{max}} = 0.02 \)
- N-UNIFORM, \( d_{\text{max}} = 0.01 \)
- N-UNIFORM, \( d_{\text{max}} = 0.005 \)

<table>
<thead>
<tr>
<th>Memory [MB]</th>
<th>( L_{\text{gap}}/d_{\text{min}} ) or ( L_{\text{gap}}/d_u )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10^1</td>
<td>2 4 6 10 20 32 64 128</td>
</tr>
</tbody>
</table>
Results

\( a = 0.09, \quad b = 0.89 \)

\[
\begin{align*}
\nu_{\text{incl}} &= -0.99 \\
\nu_{\text{incl}} &= -0.49 \\
\nu_{\text{incl}} &= 0.00 \\
\nu_{\text{incl}} &= 0.49
\end{align*}
\]

\( \nu_{\text{matr}} = 0.0 \)
Results

\[ a = 0.09, \ b = 0.89 \]

\[ \nu_{\text{incl}} = -0.99 \]
\[ \nu_{\text{incl}} = -0.49 \]
\[ \nu_{\text{incl}} = 0.00 \]
\[ \nu_{\text{incl}} = 0.49 \]

\[ \nu_{\text{matr}} = 0.0 \]

\[ \frac{E_{\text{eff}}}{E_{\text{matr}}} \]

\[ 10^{-8} \quad 10^{-6} \quad 10^{-4} \quad 10^{-2} \quad 10^{0} \quad 10^{2} \quad 10^{4} \quad 10^{6} \]

\[ 10^{-6} \quad 10^{-4} \quad 10^{-2} \quad 10^{0} \quad 10^{2} \quad 10^{4} \quad 10^{6} \]

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Results

Planar auxeticity from various inclusions

\[ a = 0.09, \ b = 0.89 \]

\[ \nu_{\text{incl}} = -0.99 \]
\[ \nu_{\text{incl}} = -0.49 \]
\[ \nu_{\text{incl}} = 0.00 \]
\[ \nu_{\text{incl}} = 0.49 \]

\[ \nu_{\text{matr}} = 0.0 \]
Results

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure}
\caption{A graph showing the relationship between $\nu$ and $E_{\text{incl}}/E_{\text{matr}}$. The inset image shows $a = 0.09$, $b = 0.89$.}
\end{figure}
Results
The dependence of Poisson’s ratio as a function of $a/b$ for various values of $L_{\text{gap}}$. Here both $\nu_{\text{matr}}$ and $\nu_{\text{incl}}$ are equal 0.
The dependence of Poisson’s ratio as a function of \( a/b \) for various values of \( L_{\text{gap}} \). Here both \( \nu_{\text{matr}} \) and \( \nu_{\text{incl}} \) are equal 0.
Rigid inclusions, \( f(L_{\text{gap}}) \)

**Figure:** The dependence of Poisson’s ratio as a function of \( L_{\text{gap}} \) for various values of \( a/b \) within the rigid inclusion limit. Here both \( \nu_{\text{matr}} \) and \( \nu_{\text{incl}} \) are equal 0.
Rigid inclusions, $f(\nu_{\text{matr}})$

Figure: The dependence of effective Poisson’s ratio as a function of the matrix Poisson’s ratio in the limit of infinitely hard inclusions ($E_{\text{incl}} \to \infty$). The value of $\nu_{\text{incl}}$ in this limit has no impact on the effective properties since the inclusions do not deform. The inset figure helps in estimating of the $a/b$ ratio, for which $\nu_{\text{eff}}$ changes sign.
More and more auxetic constituents

\begin{figure}
\centering
\includegraphics[width=\textwidth]{poisson_young}
\caption{Effective Poisson's ratio, $\nu_{\text{eff}}$, (blue colour) and effective Young's modulus, $E_{\text{eff}}/E_{\text{matr}}$, (red colour) as functions of $E_{\text{incl}}/E_{\text{matr}}$ for strongly auxetic matrices and inclusions.}
\end{figure}
Effective Poisson’s ratio, $\nu_{\text{eff}}$, and effective Young’s modulus, $E_{\text{eff}}/E_{\text{matr}}$ as functions of nominal strain for $\nu_{\text{matr}} = 0$ and “void” inclusions (the material of inclusions is entirely removed). The numbers in the legend correspond to the following geometry parameter pairs: 1) $a = 0.09, 0.89$, 2) $a = 0.14, 0.84$, 3) $a = 0.19, 0.79$, 4) $a = 0.24, 0.74$, 5) $a = 0.29, 0.69$, 6) $a = 0.34, 0.64$, 7) $a = 0.39, 0.59$, 8) $a = 0.44, 0.54$, 9) $a = 0.49, 0.49$. 
Effective Poisson’s ratio, $\nu_{\text{eff}}$, and effective Young’s modulus, $E_{\text{eff}}/E_{\text{matr}}$ as functions of nominal strain for $\nu_{\text{matr}} = 0$ and “void” inclusions (the material of inclusions is entirely removed). The numbers in the legend correspond to the following geometry parameter pairs: 1) $a = 0.09, 0.89$, 2) $a = 0.14, 0.84$, 3) $a = 0.19, 0.79$, 4) $a = 0.24, 0.74$, 5) $a = 0.29, 0.69$, 6) $a = 0.34, 0.64$, 7) $a = 0.39, 0.59$, 8) $a = 0.44, 0.54$, 9) $a = 0.49, 0.49$. 

Poisson’s ratio

Young’s modulus
Star-like inclusions, parameters $a$ and $b$

$a = 0.05, \ b = 0.945$
Star-like inclusions, parameters $a$ and $b$

\[ a = 0.05, \quad b = 0.945 \]
Conclusions

- For strongly anisotropic inclusions the effective Poisson’s ratio, which is close to $-1$ for very low $E_{incl}/E_{matr}$, **grows** and after reaching a maximum (of negative value) **decays** to the matrix values close to $E_{incl}/E_{matr} \approx 1$ and then **grows** again to the large $E_{incl}/E_{matr}$ limit.
Conclusions

- For strongly anisotropic inclusions the effective Poisson’s ratio, which is close to \(-1\) for very low $E_{\text{incl}}/E_{\text{matr}}$, grows and after reaching a maximum (of negative value) decays to the matrix values close to $E_{\text{incl}}/E_{\text{matr}} \approx 1$ and then grows again to the large $E_{\text{incl}}/E_{\text{matr}}$ limit.

- In the vicinity of $E_{\text{incl}}/E_{\text{matr}}$ for which the effective PR shows its maximum, for the most auxetic inclusions the effective Young’s modulus is essentially larger than for less auxetic inclusions.
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- In the vicinity of $E_{\text{incl}}/E_{\text{matr}}$ for which the effective PR shows its maximum, for the most auxetic inclusions the effective Young’s modulus is essentially larger than for less auxetic inclusions.

- For highly anisotropic inclusions of very large Young’s modulus, the effective PR of the composite can be negative for nonauxetic matrix and inclusions. These is a very simple example of an auxetic structure being not only entirely continuous, i.e. without empty regions (cuts, holes) introduced into the system, but with $E_{\text{incl}} \gg E_{\text{matr}}$, i.e. inclusions (much) harder than the matrix.
Thank you for your attention

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