

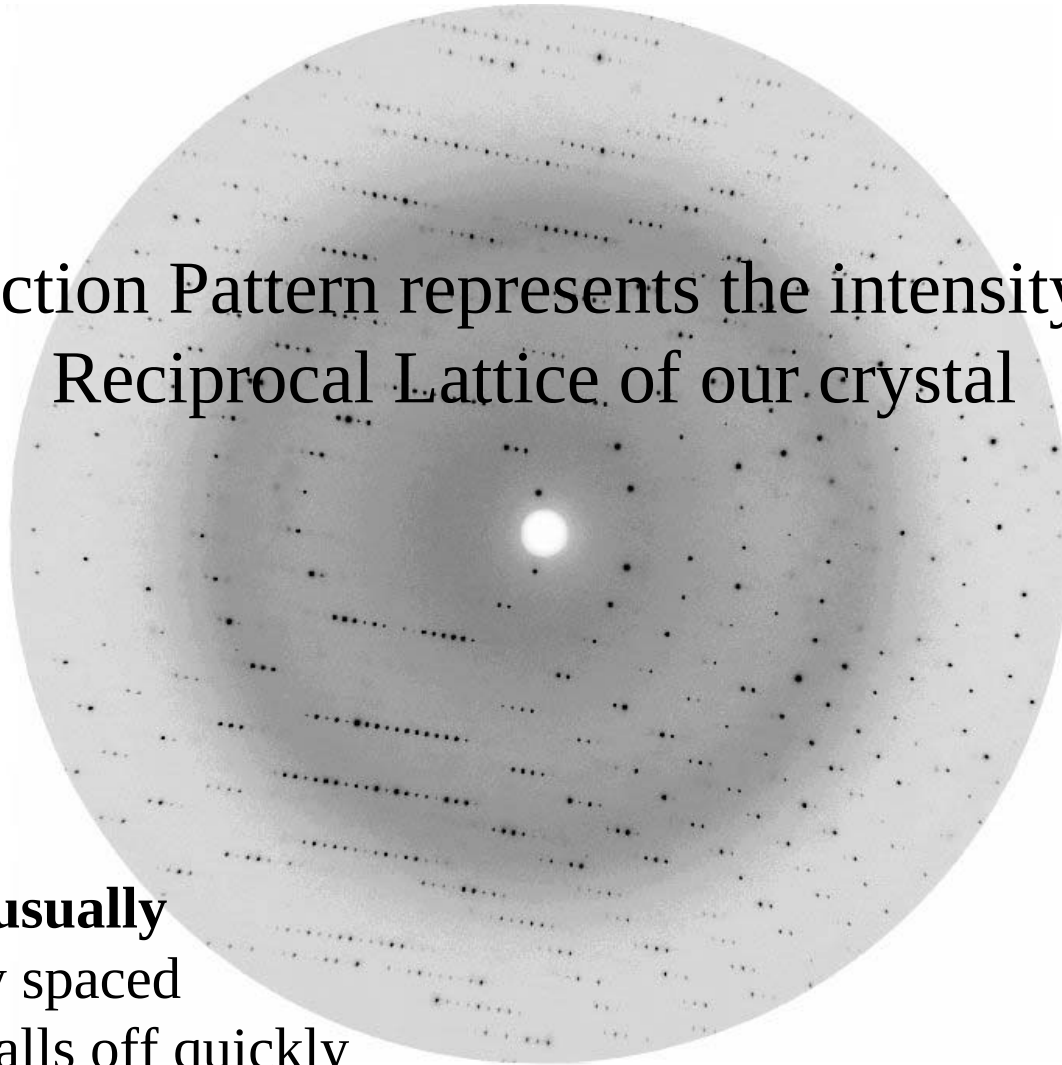
Crystallographic studies of biological macromolecules: from atomic resolution to molecular giants

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Poznan**

Protein crystal X-ray diffraction pattern

The Diffraction Pattern represents the intensity-weighted Reciprocal Lattice of our crystal



The reflections usually

- are very closely spaced
- their intensity falls off quickly
- are on strong background visible as a dark ring

The Reciprocal Lattice

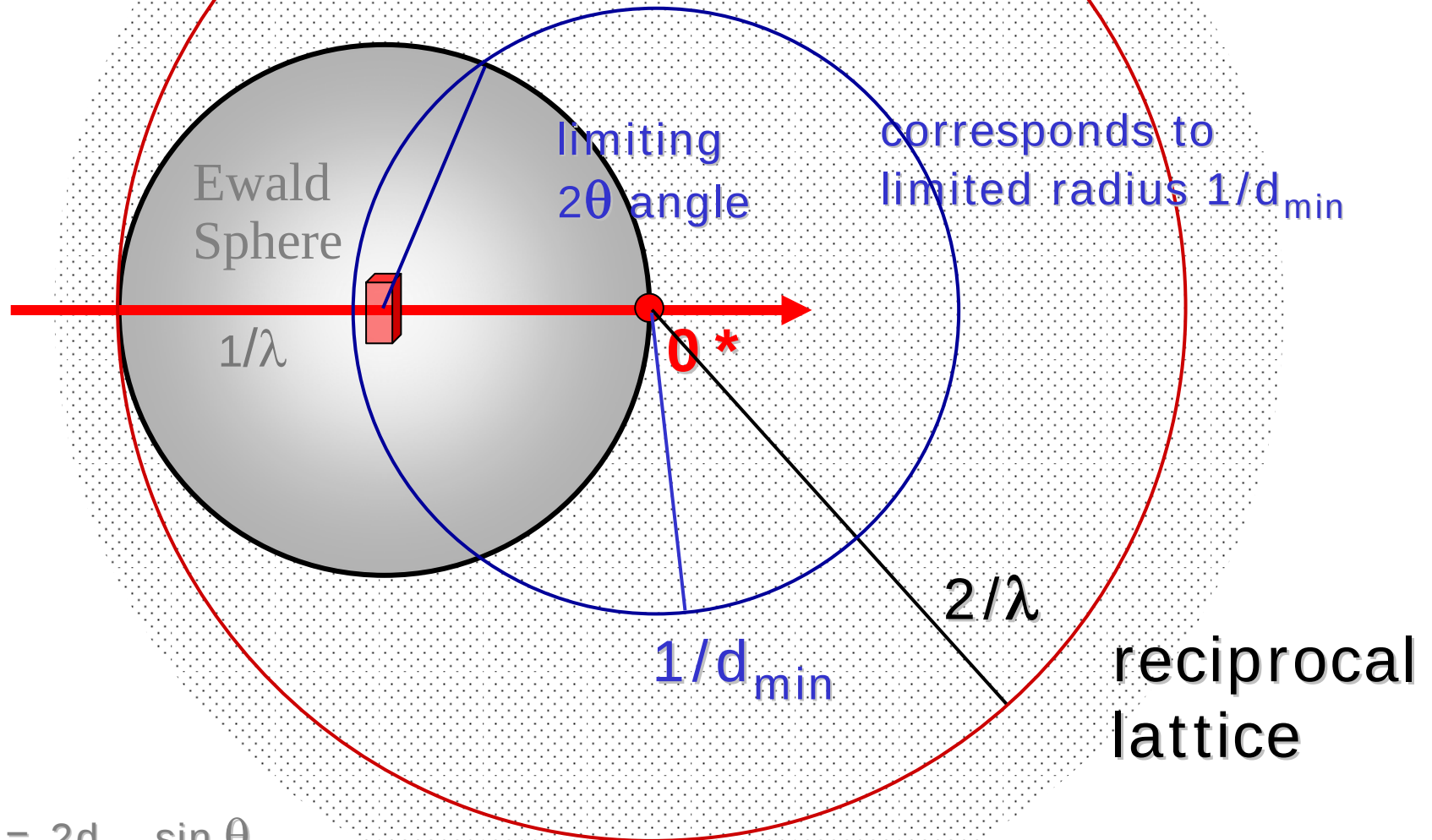
collection of points hkl associated with planes (hkl) in direct lattice

collection of nodes generated by basis vectors $\mathbf{a}^*$ $\mathbf{b}^*$ $\mathbf{c}^*$ defined by:

- direct-lattice planes (100) (010) (001)
- vector products in direct lattice, e.g. $\mathbf{c}^* = (\mathbf{a} \times \mathbf{b})/V$
- mixed scalar products: $\mathbf{a}_i \cdot \mathbf{a}_j^* = \delta_{ij}$

limiting sphere

$d_{\min} = \text{resolution limit}$



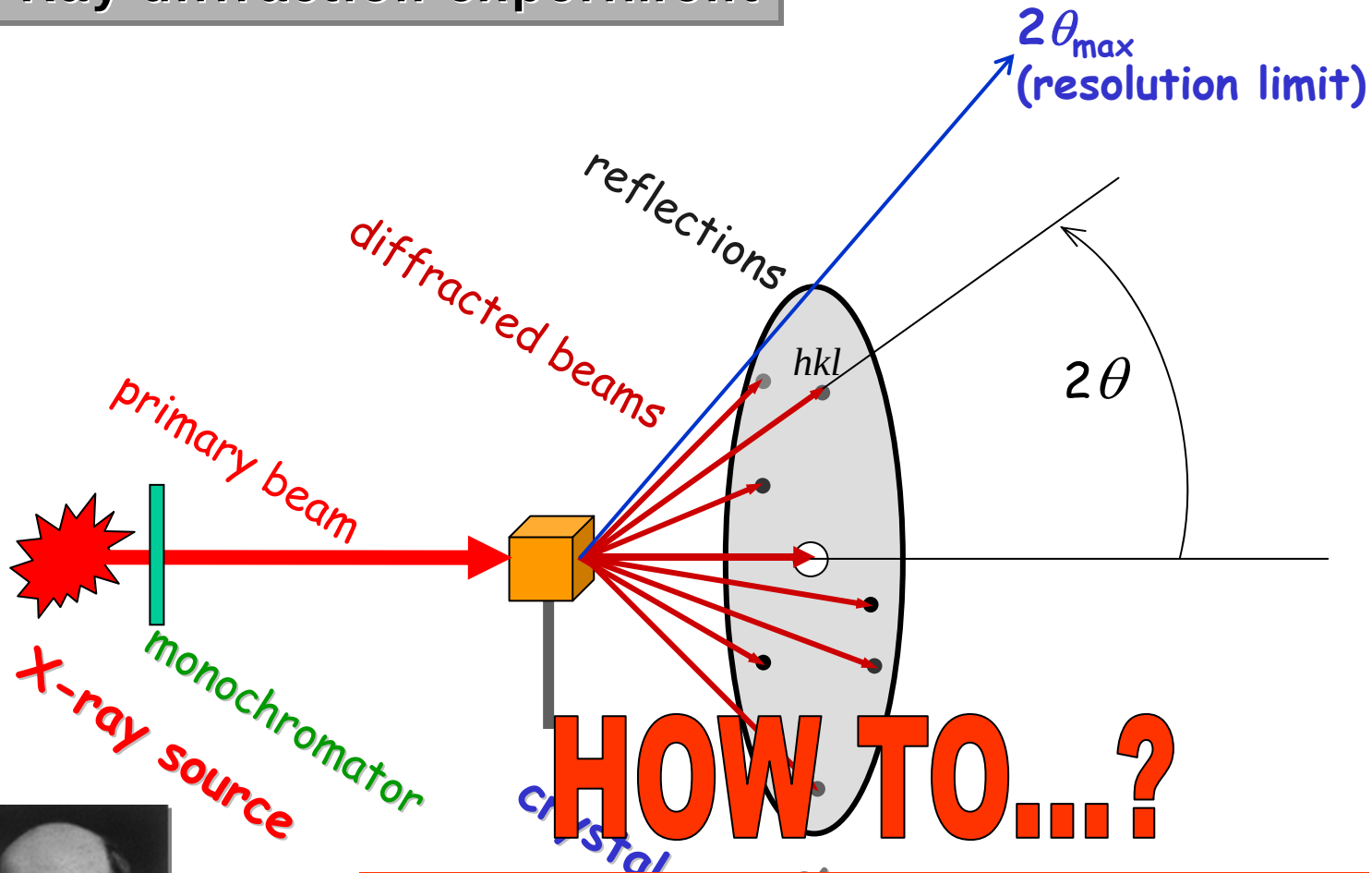
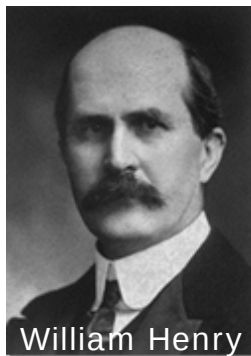
$$\lambda = 2d_{hkl} \sin \theta$$

X-Ray diffraction experiment

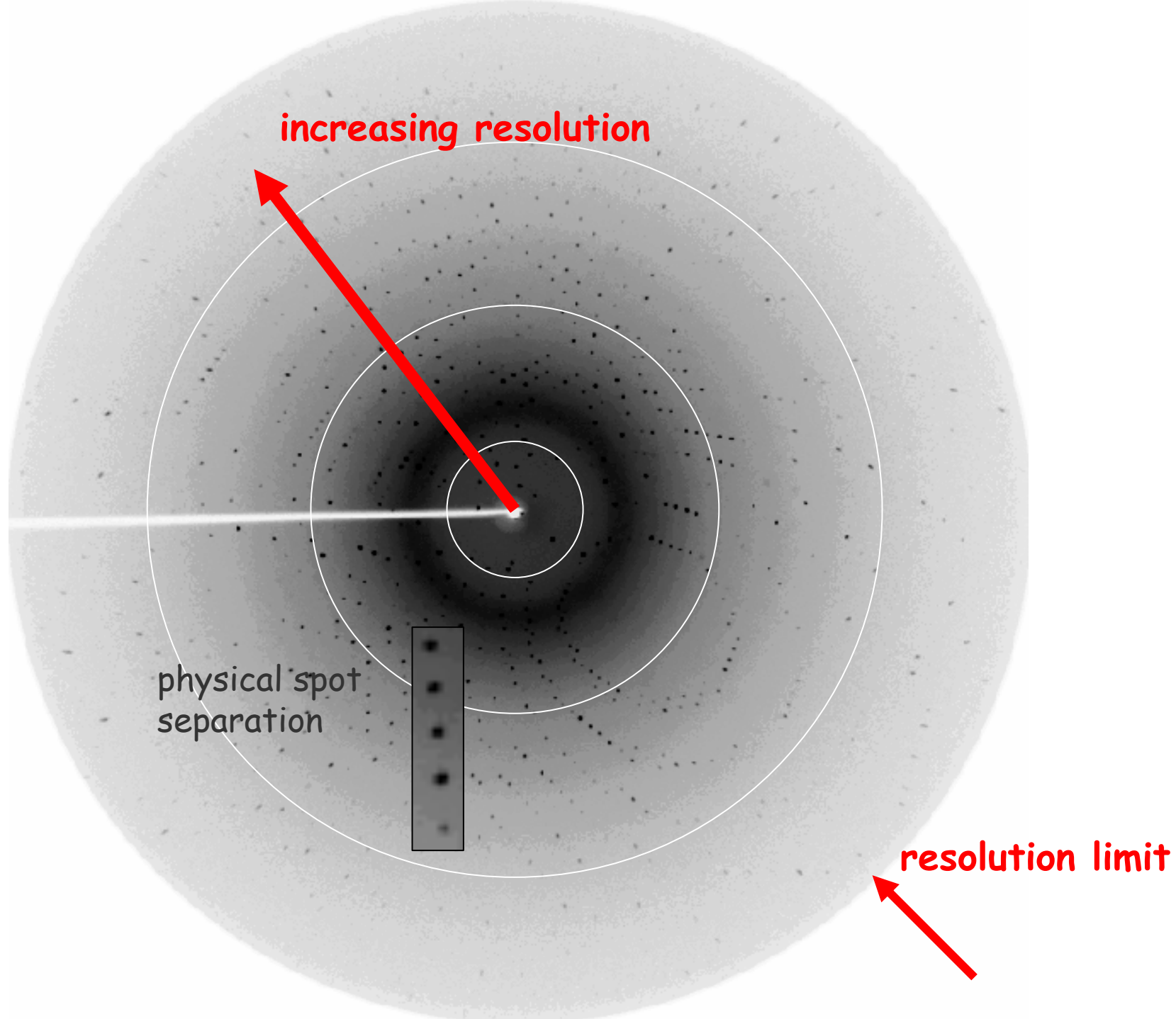


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$$\lambda = 2d_{hkl} \sin \theta$$



- capture more reflections on the detector
- change separation between reflections



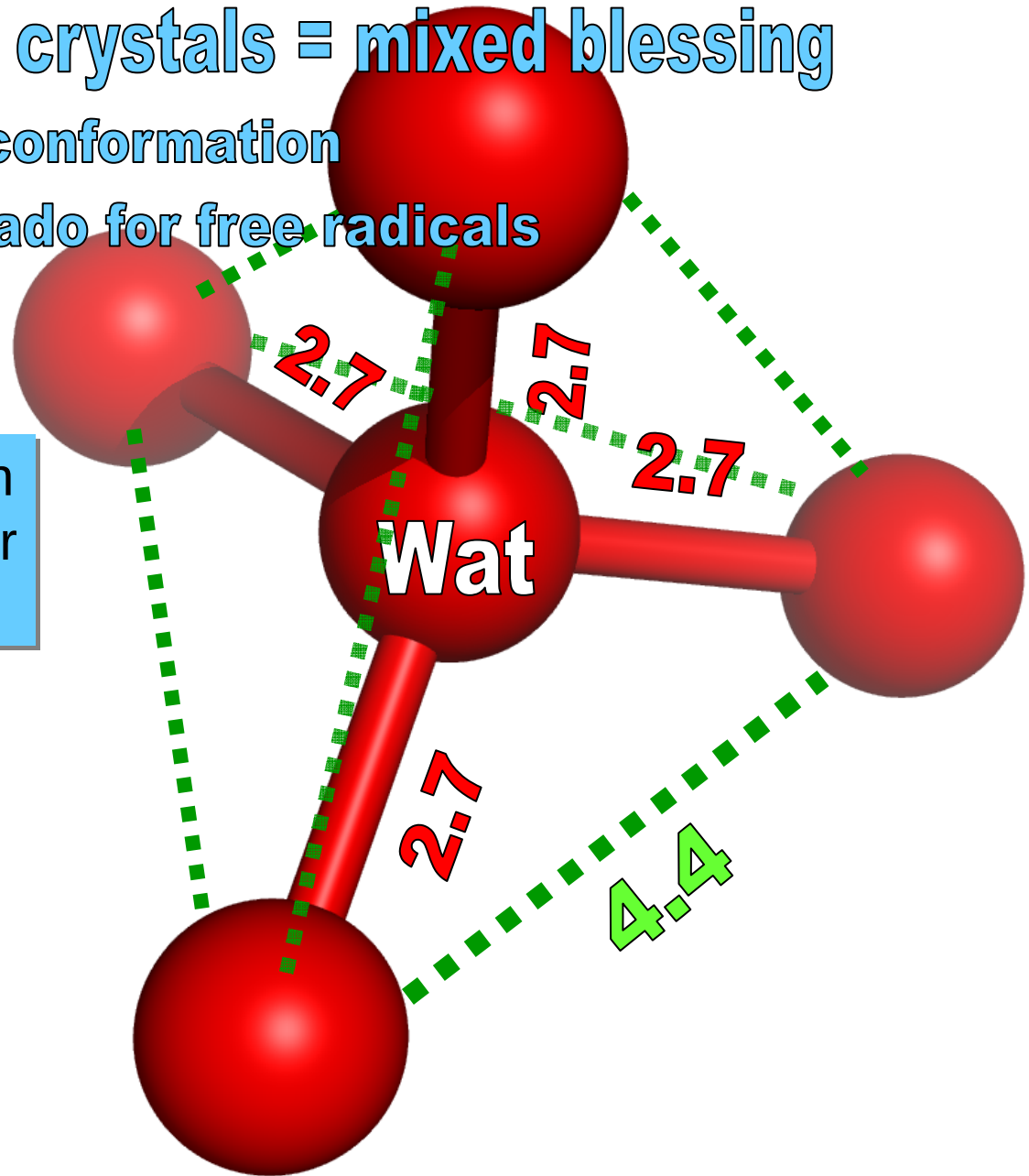
Water in protein crystals = mixed blessing

+ native protein conformation

- disorder, El Dorado for free radicals

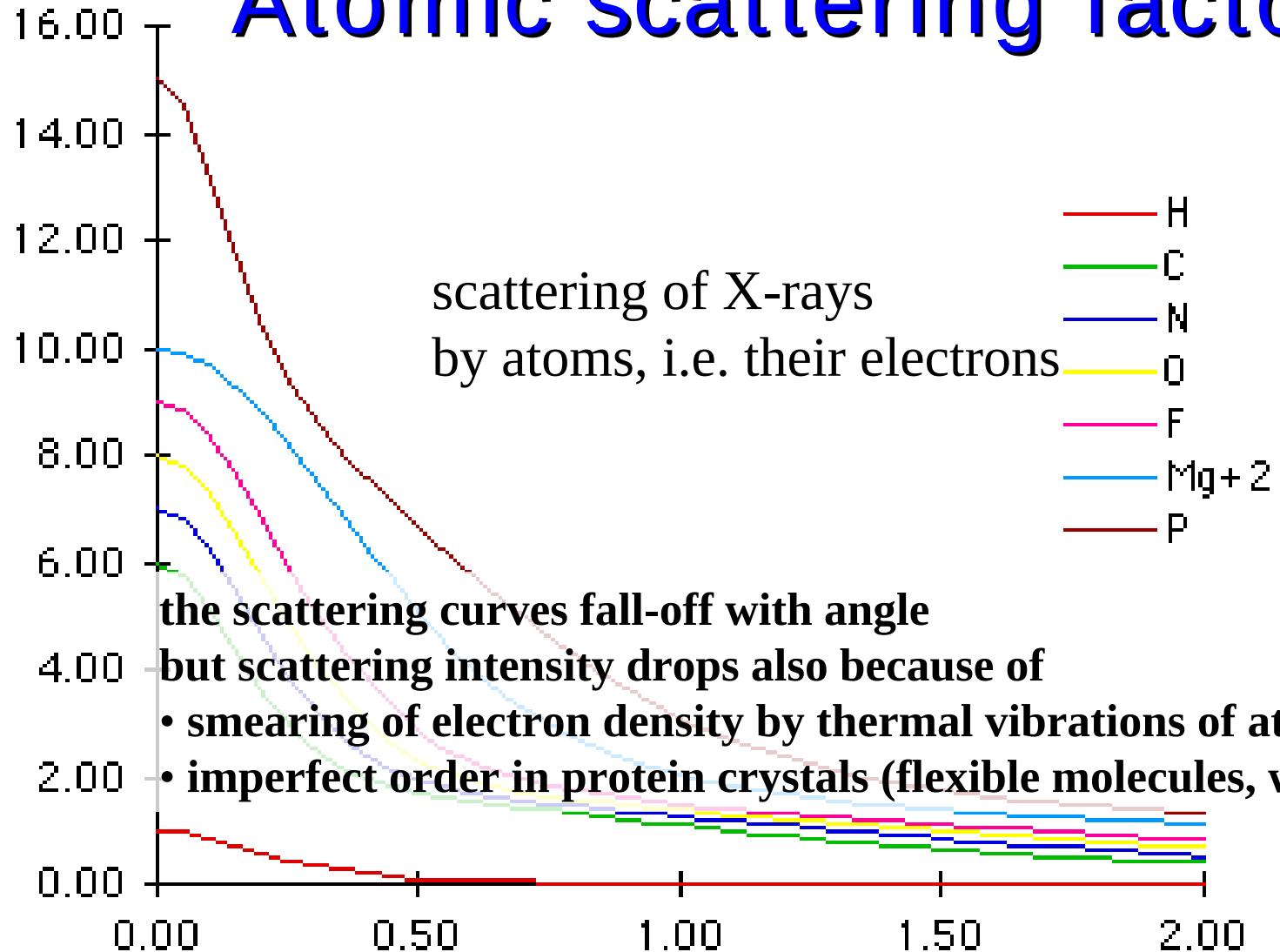
Protein crystals contain a large volume of water (about 50%)

Water molecule with four neighbors H-bonded in a tetrahedral fashion. Various oriented 1-3 O...O distances 4.4 Å long will abound.



Atomic scattering factors

e



the scattering curves fall-off with angle

but scattering intensity drops also because of

- smearing of electron density by thermal vibrations of atoms
- imperfect order in protein crystals (flexible molecules, water)

$\sin\theta/\lambda$



Jean Baptiste Joseph Fourier
1768-1830

The relation between
electron density distribution
(= crystal structure)
and
the X-ray diffraction pattern
is governed by
Fourier Transformation

To be able to determine this electron distribution
- or electron density -
(in the form of electron density maps)
is of great importance to us:
in chemistry – everything is explained by electrons:
the character of different atoms
and the bonds between them in molecules

Scattering of X-rays by matter

- X-rays are scattered by electrons
- the Amplitude of scattering is proportional to the number of electrons

$$F(\mathbf{r}^*) = \int_V \rho(\mathbf{r}) \exp[2\pi i(\mathbf{r}^* \cdot \mathbf{r})] d\mathbf{r}$$

ρ is the density of electrons at point $\mathbf{r}$ within a volume V

$\mathbf{r}^*$ is a point in the reciprocal space defining the direction of scattering

$F(\mathbf{r}^*)$ is called the Structure Factor

- the Intensity of scattering is proportional to squared Amplitude

$$I(\mathbf{r}^*) \sim F^2(\mathbf{r}^*)$$

The Fourier Transform

the expression

$$\mathbf{F}(\mathbf{r}^*) = \int_V \rho(\mathbf{r}) \exp[2\pi i(\mathbf{r}^* \cdot \mathbf{r})] d\mathbf{r}$$

is at the heart of a mathematical theory formulated by
Jean Baptiste Joseph Fourier (1768-1830)

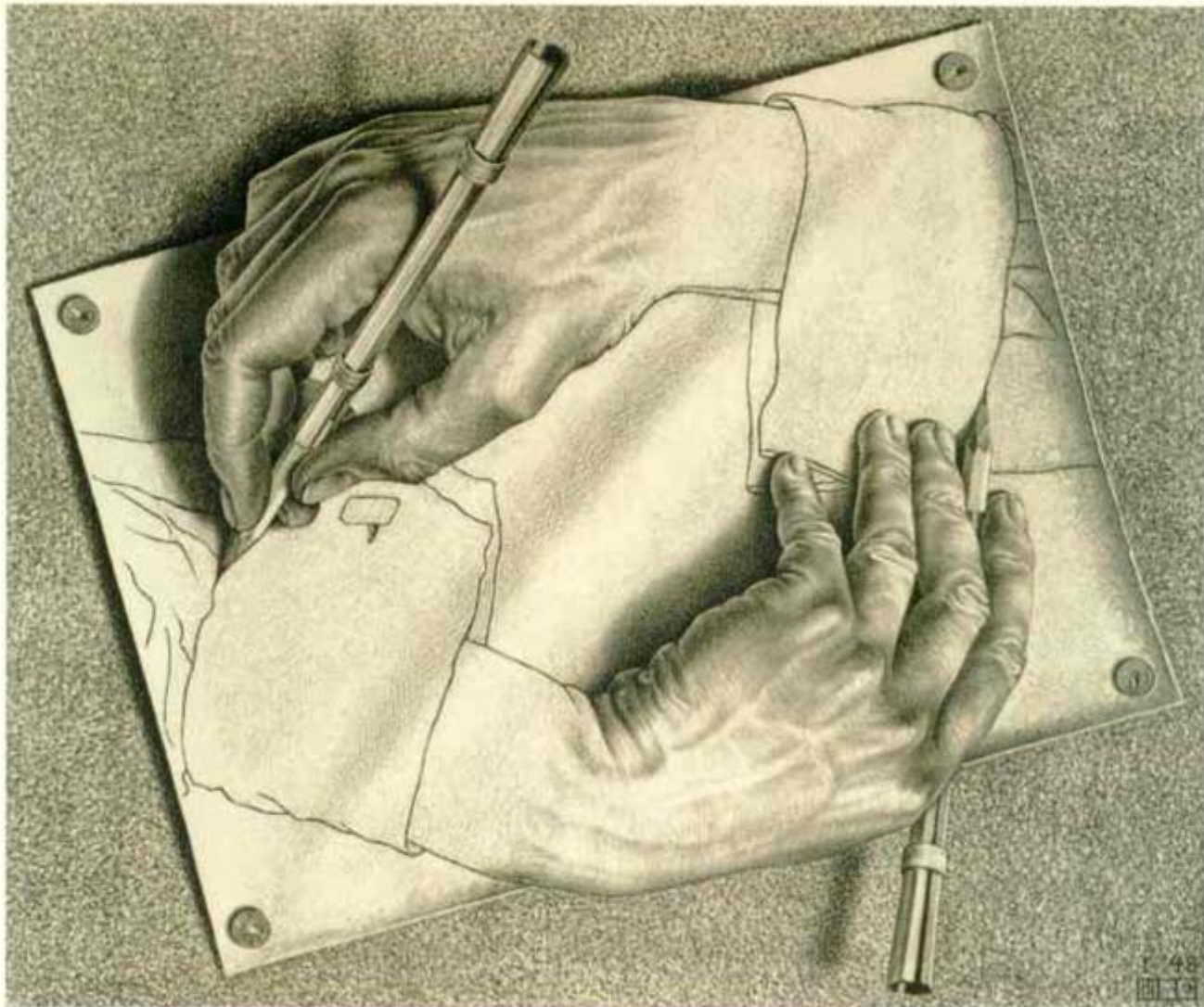
It defines, that the (complex) function

$\mathbf{F}(\mathbf{r}^*)$ is the Fourier Transform of the function $\rho(\mathbf{r})$

Simultaneously, an almost identical expression defines
 $\rho(\mathbf{r})$ *in terms of* **$\mathbf{F}(\mathbf{r}^*)$**

$$\rho(\mathbf{r}) = \int_{V^*} \mathbf{F}(\mathbf{r}^*) \exp[-2\pi i(\mathbf{r}^* \cdot \mathbf{r})] d\mathbf{r}^*$$

Fourier Transform

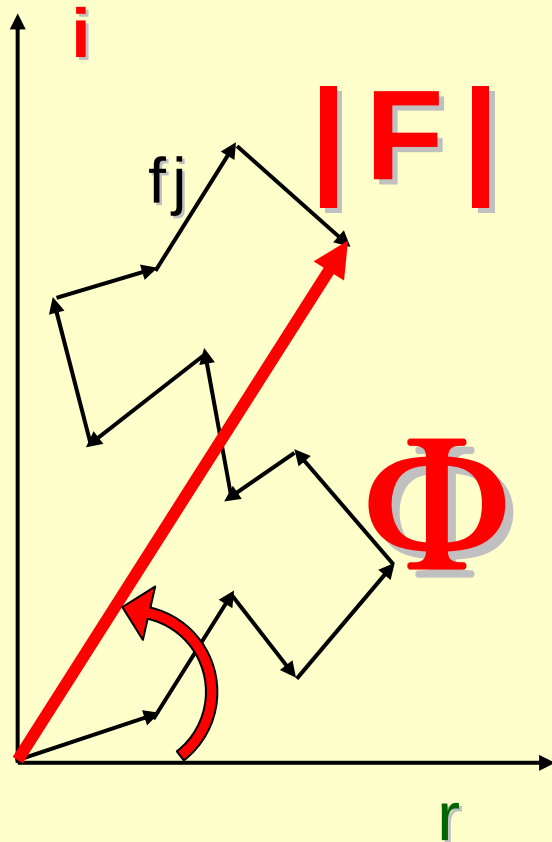


Maurits Cornelis Escher (1898-1972)

Drawing Hands

1948

**Structure Factor $F^2(hkl)$ = Intensity
and electron density $\rho(xyz)$**



$$F(hkl) = \sum f_j \exp[2\pi i(hx_j + ky_j + lz_j)]$$

$$\rho(xyz) = V^* \sum F(hkl) \exp[-2\pi i(hx + ky + lz)]$$

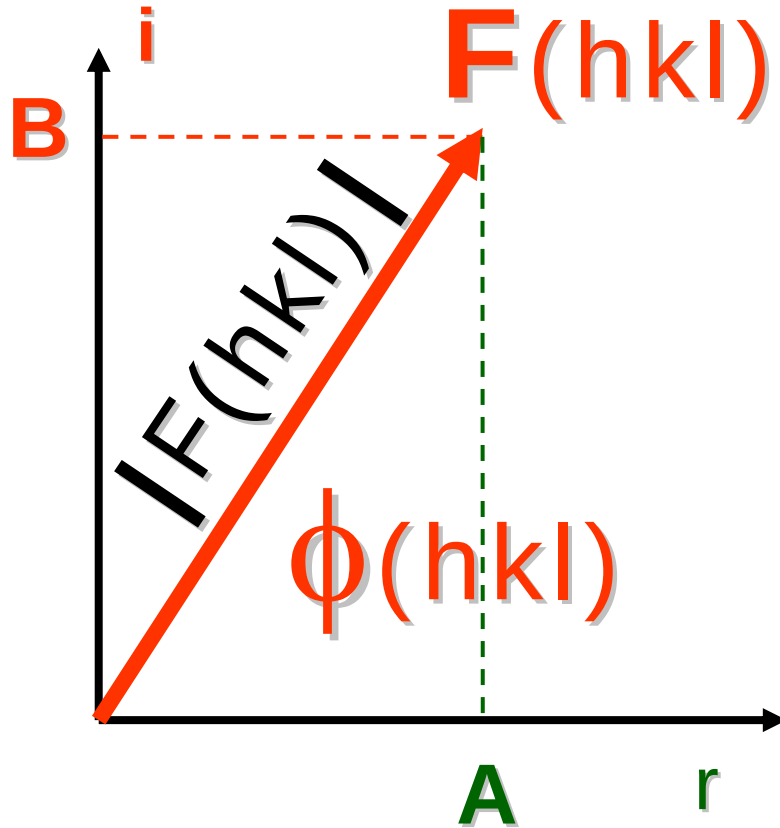
Fourier Transforms

$$F(hkl) = |F| \cdot \exp(i\phi)$$

$$|F(hkl)| = \sqrt{I(hkl)}$$

The Φ ase Problem

The Structure Factor is a complex entity



$$\begin{aligned}
 F(hkl) &= A + iB \\
 &= |F(hkl)| \cdot (\cos\phi + i\sin\phi) \\
 &= |F(hkl)| \cdot \exp(i\phi)
 \end{aligned}$$



magnitude

Euler
notation



phase

The Patterson Function



- Fourier transform of the squares of the structure factors, i.e. of reflection intensities:

$$P(\mathbf{u}) = \sum |F(\mathbf{h})|^2 \exp[-2\pi i(\mathbf{h} \cdot \mathbf{u})]$$

- the distribution of atoms convoluted with its centrosymmetric image (autocorrelation function):

$$P(u) = \rho(x) \otimes \rho(-x) = \int \rho(x) \rho(x-u) dx$$

- the peaks represent interatomic vectors
- each pair of atoms has a unique Patterson peak
- the peak height is the product of atomic numbers
- very complex: N^2 Patterson peaks for an N-atom structure

Protein crystallographic methods for solving the **Phase Problem**

all rely to some extent on the Patterson function

- Isomorphous Replacement (MIR/SIR)
(Max Perutz)
- Molecular Replacement (MR)
(Michael Rossmann)
- Multiwavelength Anomalous Diffraction (MAD)
(Wayne Hendrickson)

An electron density map is the primary product of an X-ray diffraction study of a protein crystal.
Everything else is its interpretation.

Electron density maps are generated via Fourier Transformation.
This is only possible when suitable Phases can be attributed to the experimental Structure Factor Amplitudes

There are two fundamental ways to obtain the Phases:

Through additional
experimental measurements

in the methods of

Isomorphous Replacement (MIR) or

Multiwavelength Anomalous Diffraction (MAD)

Experimental Phases

Through
calculations

using an **assumed model**

in the method of

Molecular Replacement (MR)

Model-derived Phases
can lead to
model bias

Molecular Replacement (Michael Rossmann)

MR

- Atomic model of a similar protein

becomes a probe with which

we can solve the unknown crystal structure
of a new protein

Main steps of the method of Molecular Replacement

we must have a sufficiently similar molecular model for the unknown crystal structure

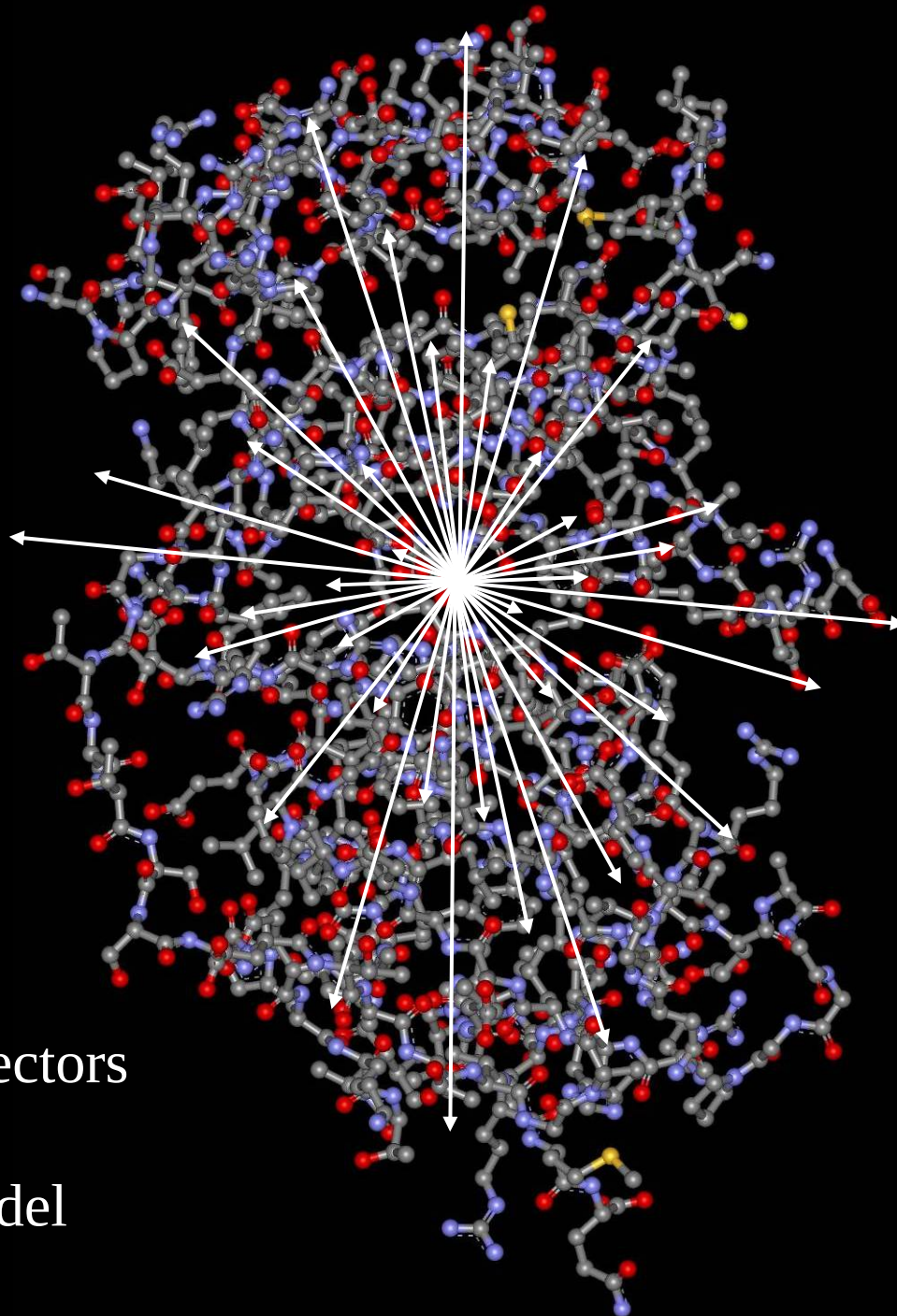
we do not know the crystal architecture, i.e. the orientation and placement of the model in the unit cell are unknown

calculate the experimental Pattersona map, P_o , based on F_o^2 , for the unknown structure

generate a set of interatomic vectors, P_m , from the model

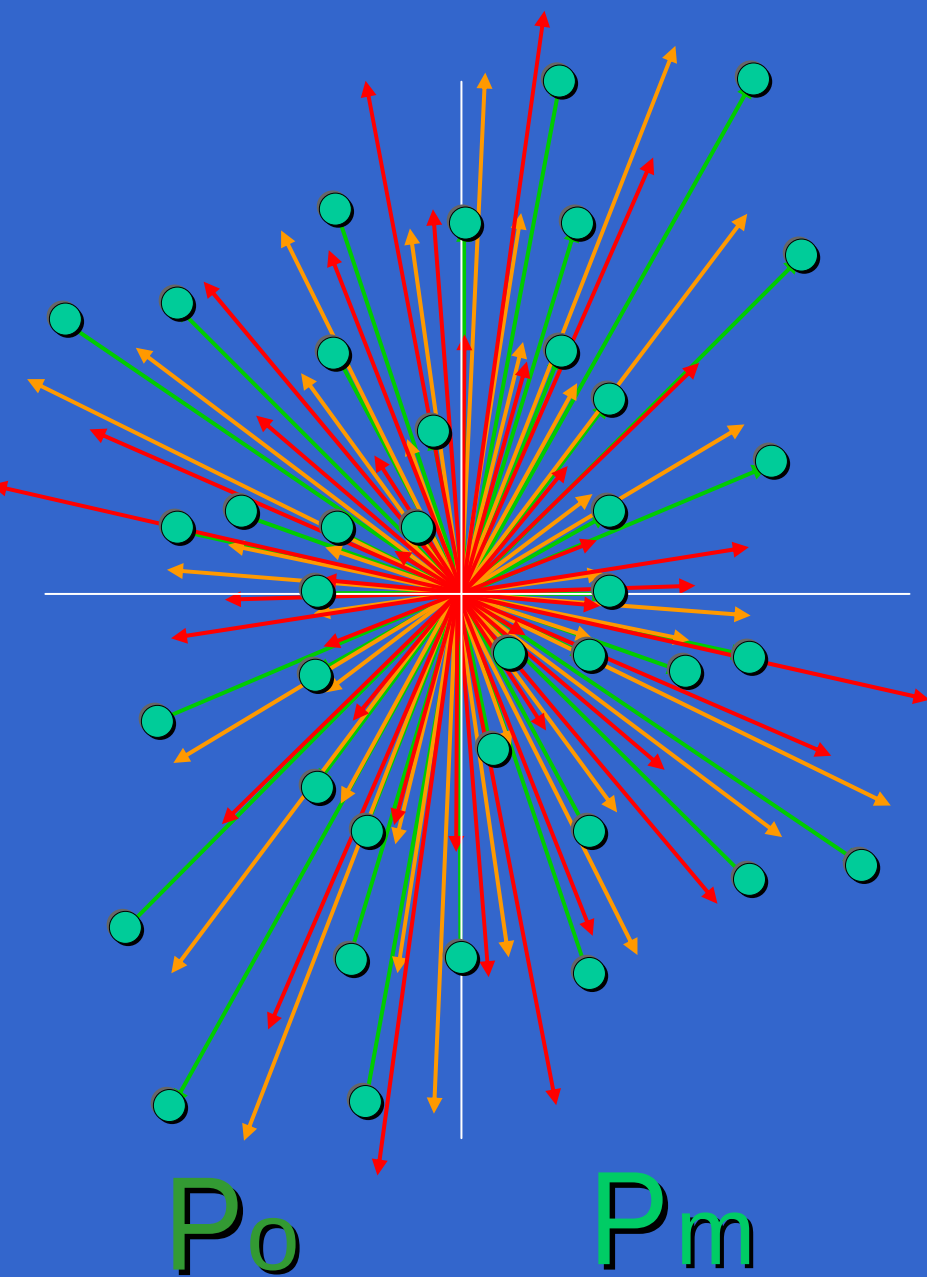
find the orientation of P_m when superposed on the P_o map

find the position of the model relative to the unit cell origin in the unknown crystal structure



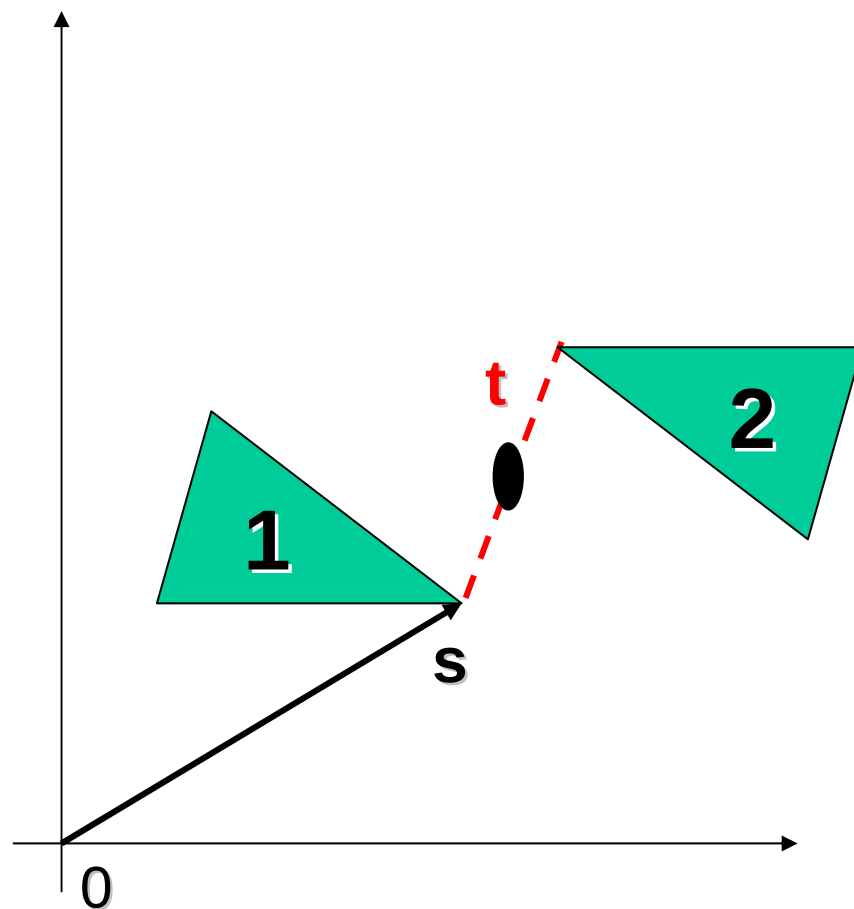
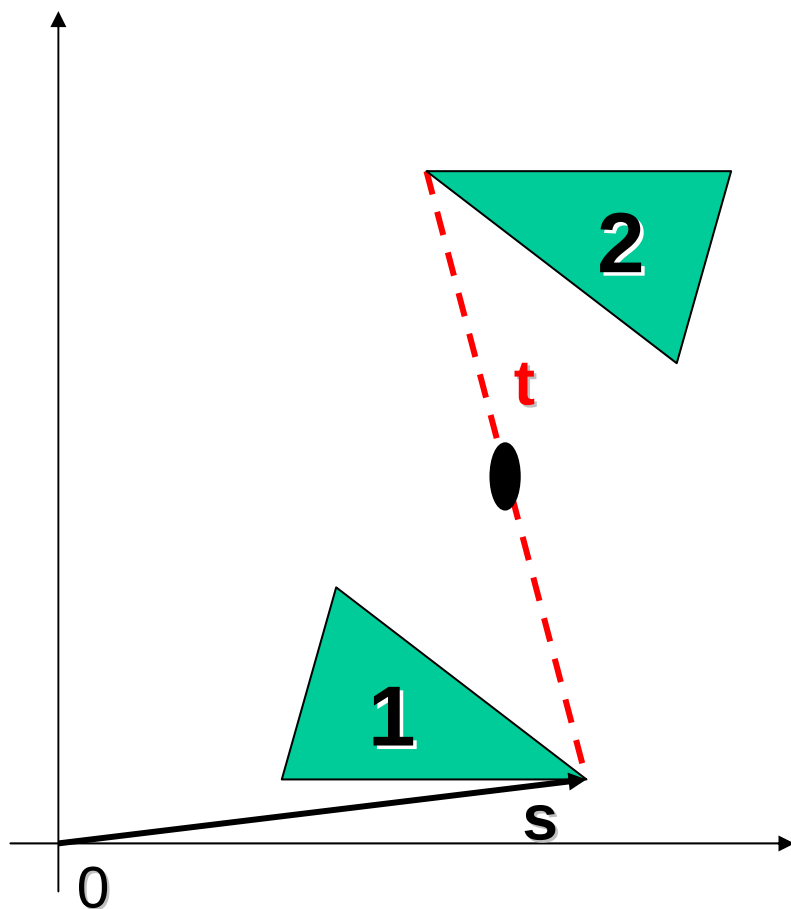
interatomic vectors
derived from
an atomic model

P_m



Solution
of the rotation
problem in
Molecular
Replacement

Diagram illustrating the solution of the rotation problem in Molecular Replacement. The diagram shows a central point from which numerous red arrows radiate outwards, representing different orientations of a molecule. The arrows are labeled P_m at the bottom, indicating the final position or orientation. The arrows are distributed in a fan-like pattern, suggesting a search for the correct orientation.



Translation s of a correctly oriented model changes the intermolecular vectors t generated by symmetry

Isomorphous Replacement (Max Perutz)

SIR / MIR

- **A few heavy atoms** in an isomorphously derivatized protein crystal become markers of electron density in the structure – they are the starting point for deciphering of the entire structure



Main steps of the Method of Isomorphous Replacement

introduce to the crystal lattice, without distorting its structure, a few electron-rich atoms

compare the diffraction of the native crystal and of the isomorphous derivative

determine the location of the heavy atoms in the structure

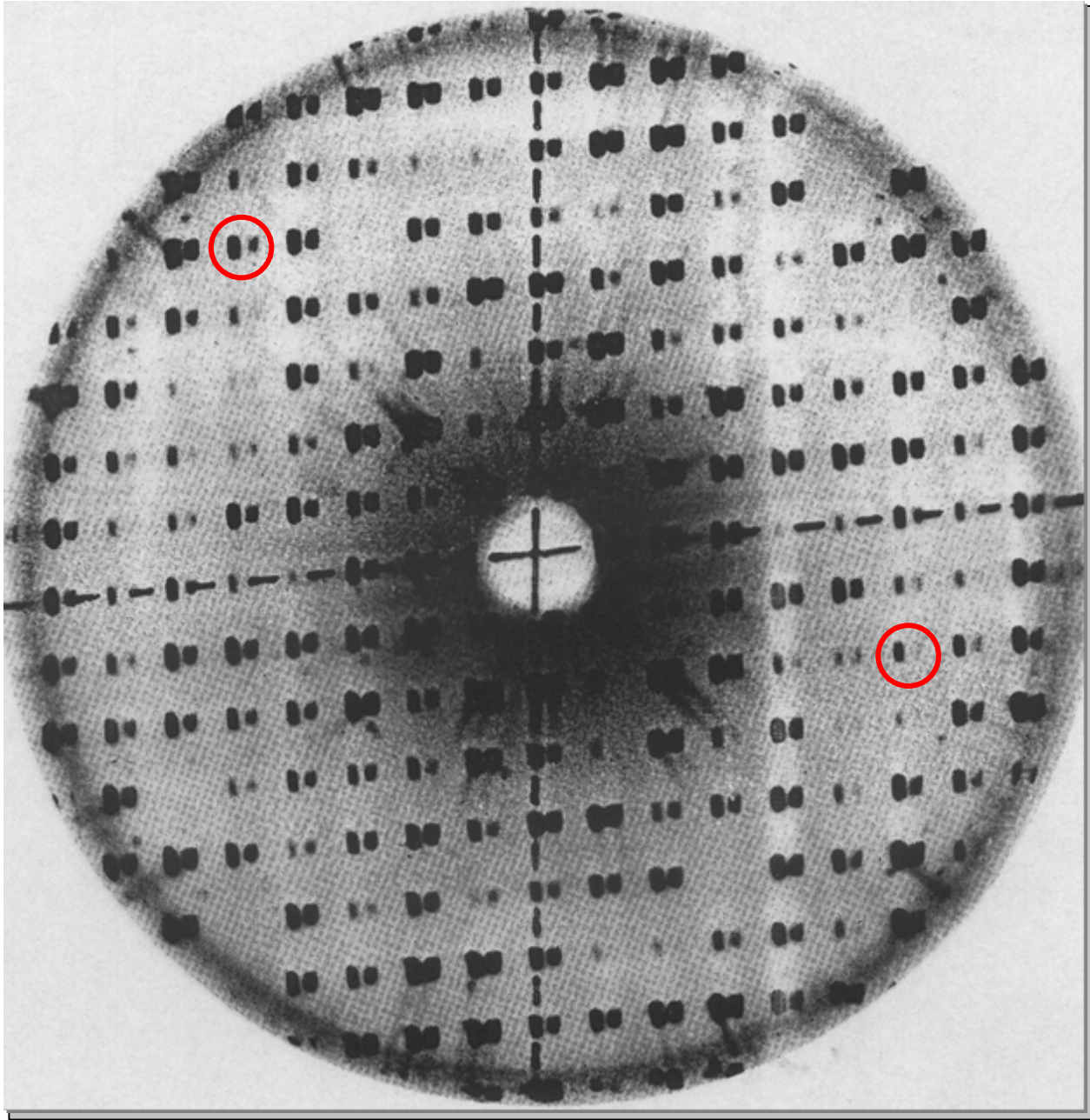
use the information about the heavy atoms to estimate the phases of the reflections

F_{PH}

Heavy atom derivative

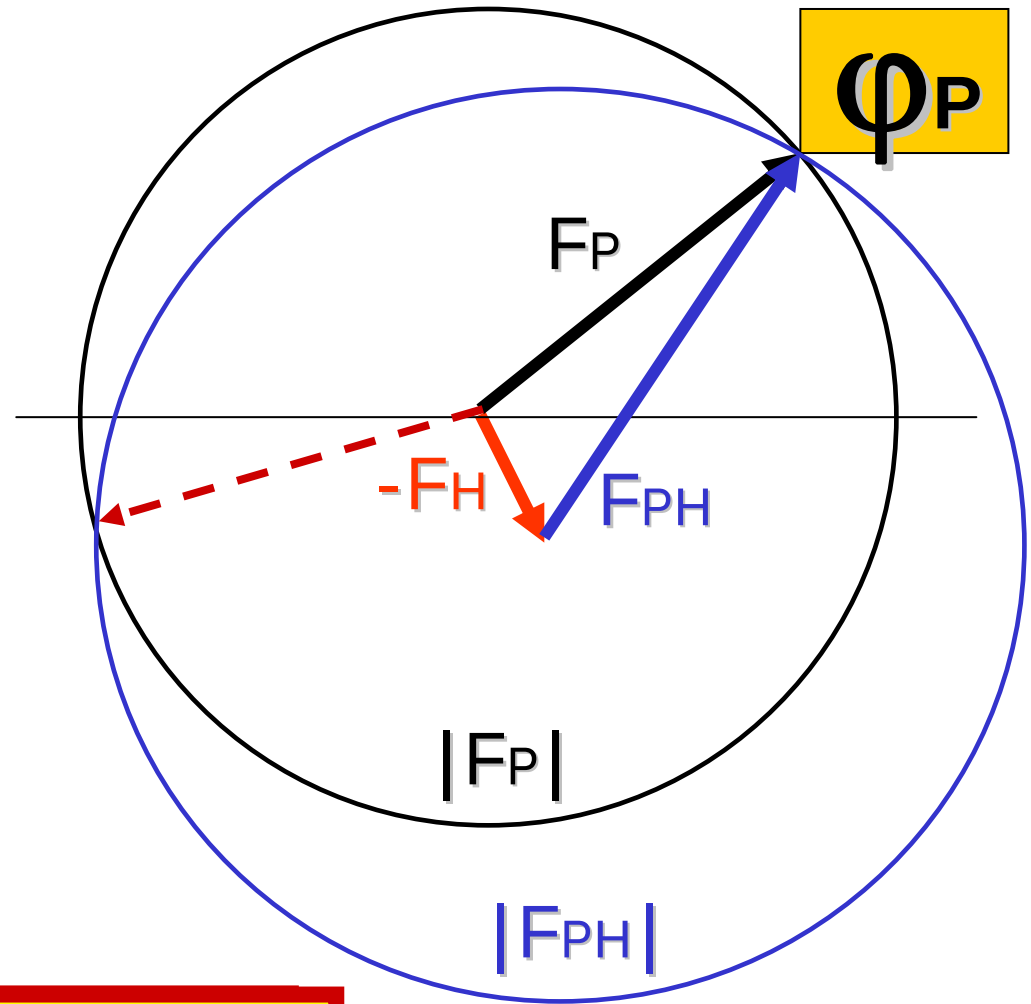
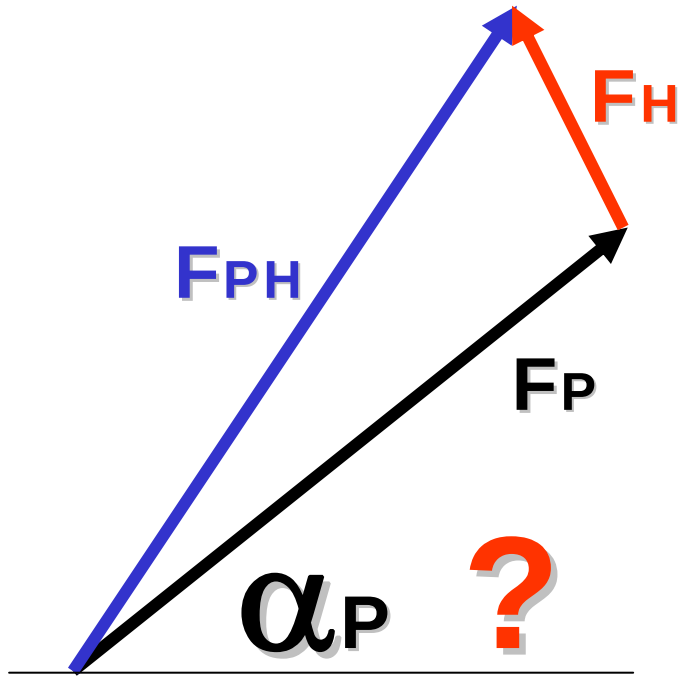
F_P

Native crystal



Two X-ray photographs superposed

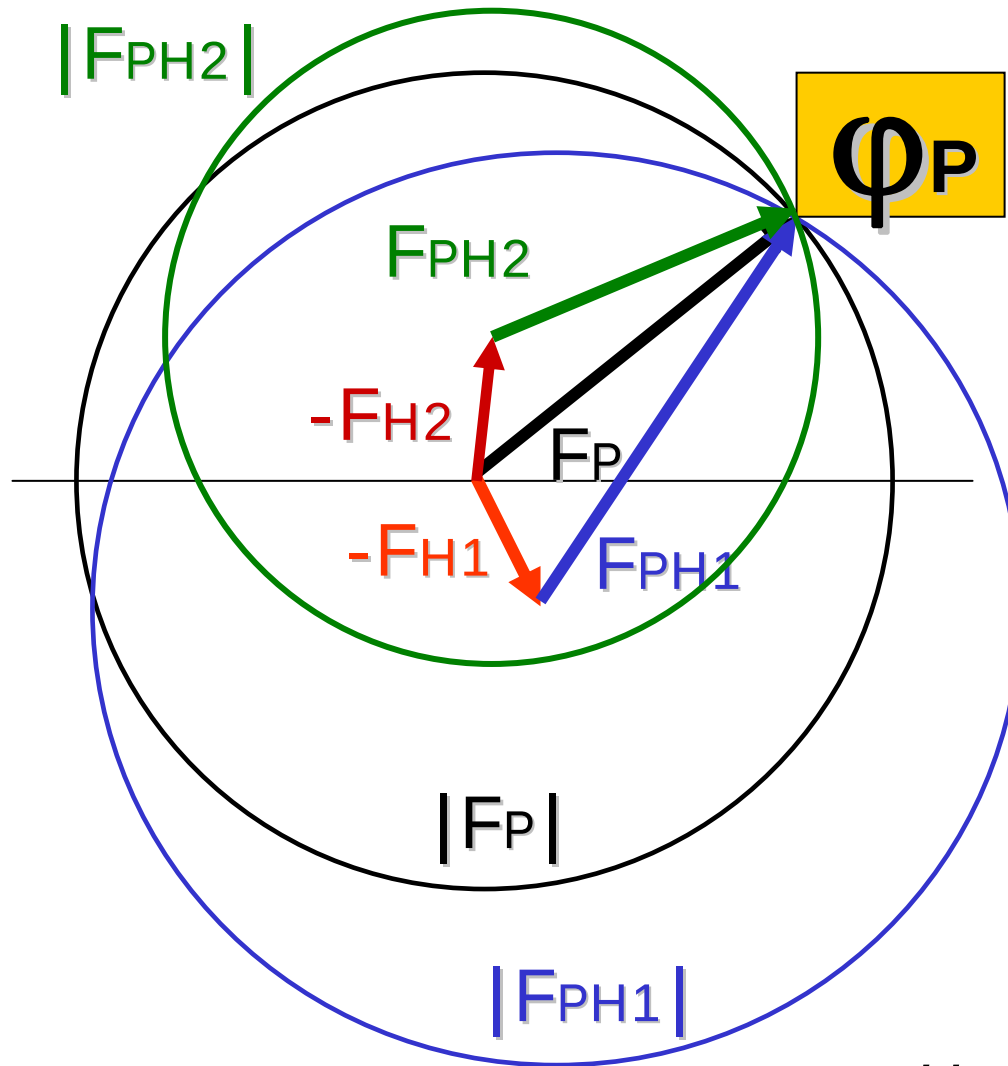
Isomorphous replacement SIR



$$-F_H + F_{PH} = F_P$$

Harker diagram

Isomorphous replacement MIR



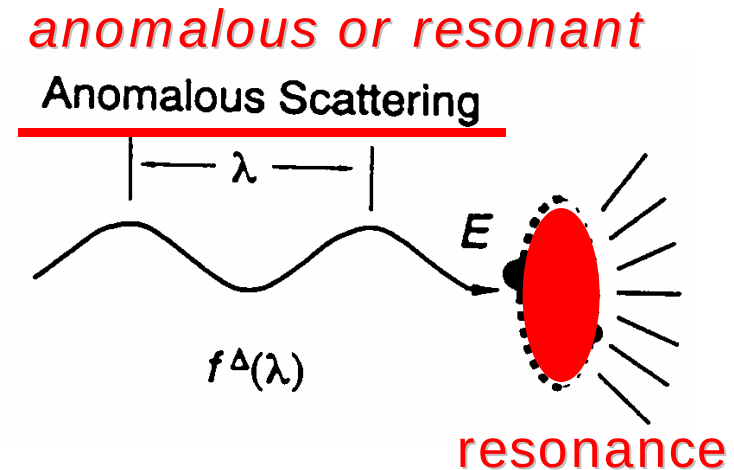
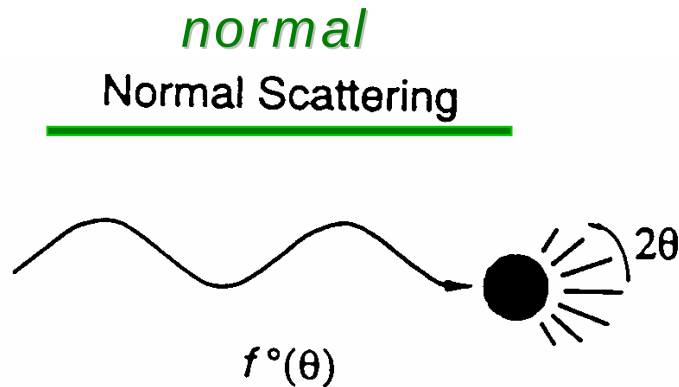
Harker diagram

MAD (Wayne Hendrickson)

- Our protein is 'labeled' with an atom (usually **Se** introduced as **Se-Met**) that can scatter X-rays anomalously
- The energy of the X-rays is tuned (**synchrotron**) to maximize the anomalous effect
- An analysis of the anomalous effects reveals the **phases**, and the phases reveal the crystal structure

MAD: *Multiwavelength Anomalous Diffraction*

Scattering of X-rays by electrons in atoms



1. depends on the scattering angle 2θ
2. does not depend on wavelength (except resonance conditions)

1. does not depend on the scattering angle 2θ
2. critically depends on wavelength

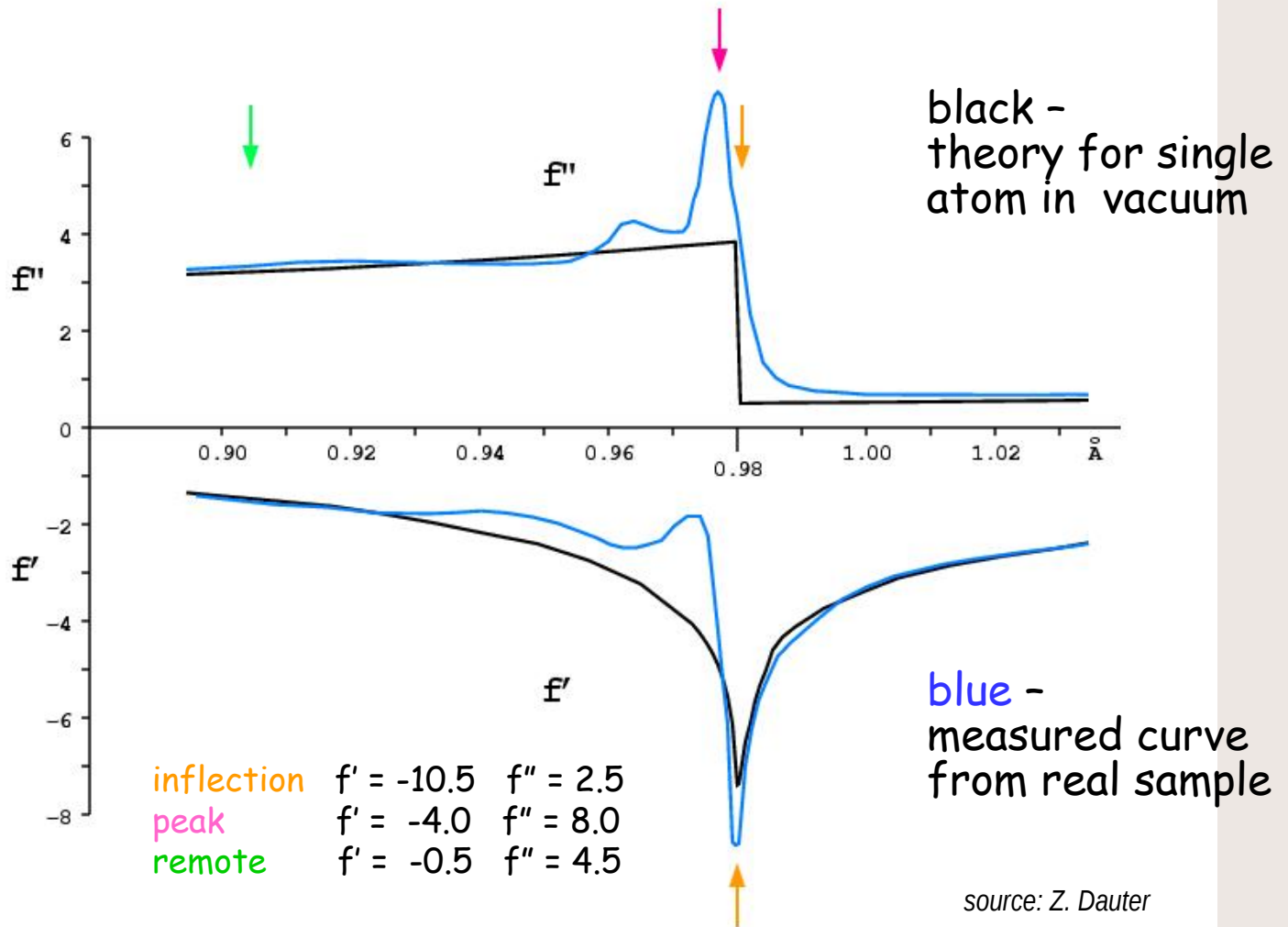
$$f_a = f_0 + f' + if''$$

dispersive
correction
(real)

Bijvoet
absorptive
correction
(imaginary)

**The atomic scattering factor
in the case of anomalous scattering**

Anomalous corrections f' and f'' for Se



Algebraic solution of the phase problem

(Jerome Karle, Wayne Hendrickson) in MAD

$$\begin{aligned} |\lambda F(h)|^2 = & |{}^0F_T|^2 + a(\lambda) |{}^0F_A|^2 \\ & + b(\lambda) |{}^0F_A| |{}^0F_T| \cos({}^0\phi_T - {}^0\phi_A) \\ & + c(\lambda) |{}^0F_A| |{}^0F_T| \sin({}^0\phi_T - {}^0\phi_A) \end{aligned}$$

separation of λ -independent:

$$(|{}^0F_T|, |{}^0F_A|, {}^0\phi_T - {}^0\phi_A)$$

and λ -dependent variables:

$$a(\lambda) = (f'^2 + f''^2)/f_0^2$$

$$b(\lambda) = 2(f'/f_0)$$

$$c(\lambda) = 2(f''/f_0)$$

Main steps of MAD

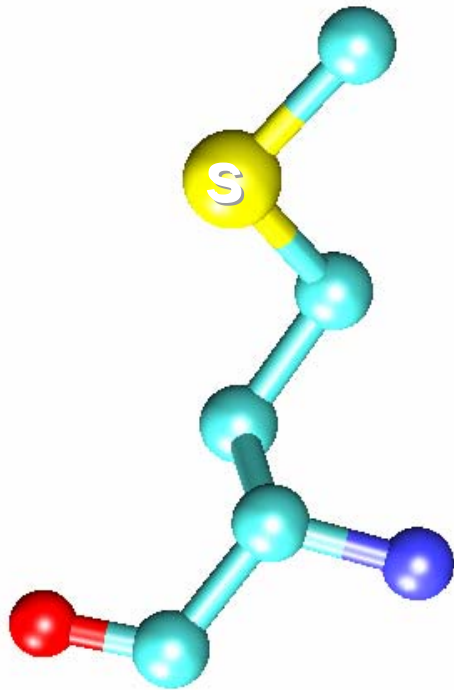
1. Elaborate an expression system of your protein in *Escherichia coli*
2. Express, purify and crystallize an Se-Met variant of your protein
3. Tune the wavelength of synchrotron radiation to resonance with Se atoms
4. Collect X-ray diffraction data at λ_{peak} , λ_{edge} and λ_{remote}
5. Solve the phase problem using the MAD algebra

Selenomethionine

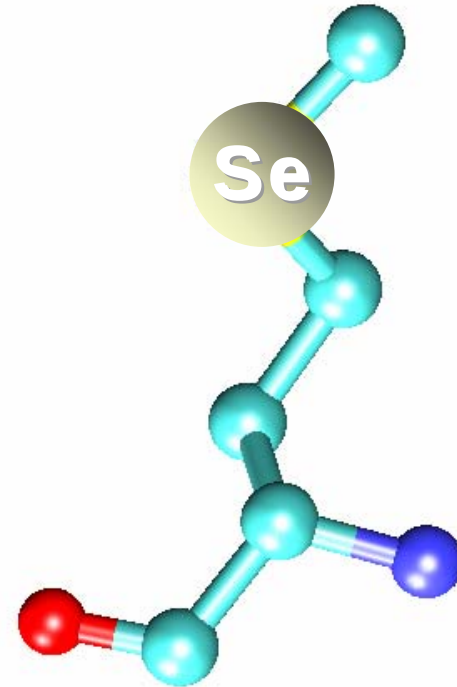
MAD

in the method of

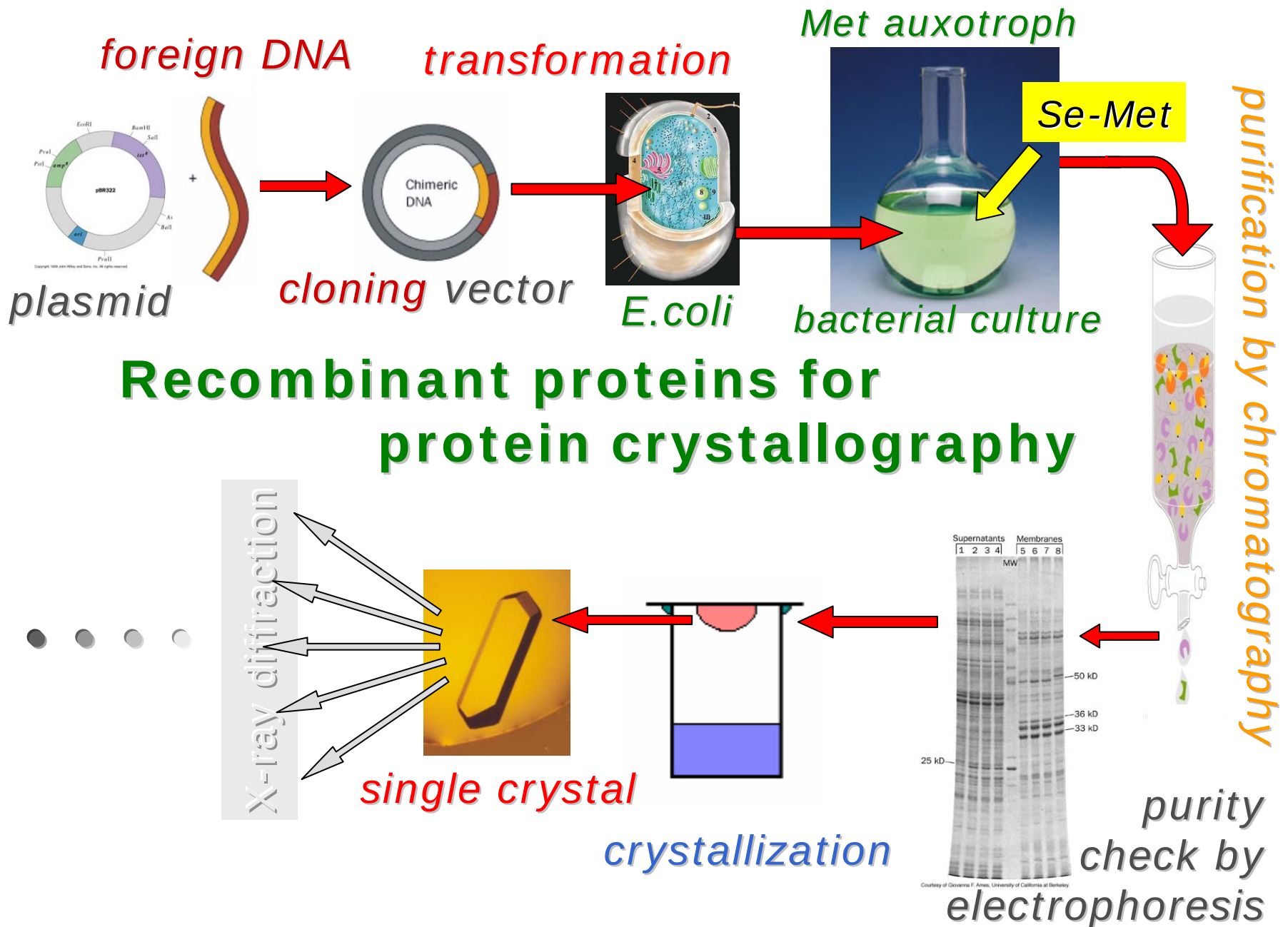
(**M**ultiwavelength **A**nomalous **D**iffraction)



Met



Se-Met



When the phase problem has been solved,
by any of the three methods,
electron density maps can be calculated
and an atomic model can be built in them.
But even if a preliminary model of our protein
has been built,
it still needs to be **refined**.

Structure Refinement

- Minimization of:

$$S1 = \sum w (|F_o| - |F_c|)^2$$

- Adjustable parameters:

for each atom, its coordinates x y z (position) and
Atomic Displacement Parameter B (amplitude of vibration)

- Gigantic number of model parameters! *e.g.* a 50 kDa protein -
ca 450 residues - *ca* 3500 non-H atoms - *ca* 14 000 parameters!
At 3 Å resolution only *ca* 8500 reflections!

Solution: get more experimental data (always the best) or
reduce the number of parameters (constraints) or
better increase the number of "observations" (restraints)

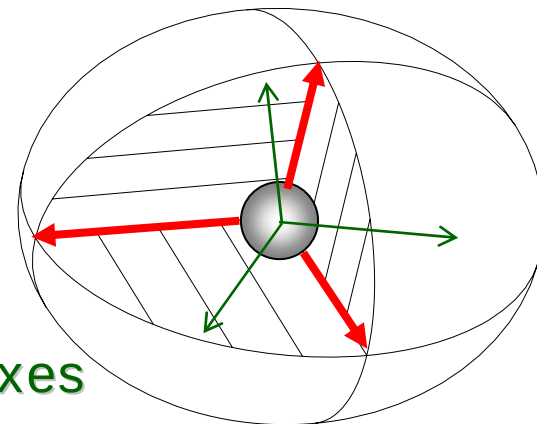
- Stereochemical restraints as additional equations:

$$S2 = \sum w [p(\text{ideal}) - p(\text{calc})]^2$$

(analogy to "energy" minimization)

The Temperature Factor or **ADP** **Atomic Displacement Parameter**

anisotropic thermal ellipsoid
expressed in crystallographic axes



Atomic vibrations (displacements from equilibrium positions) and **disorder** smear the electron density.

The vibrations are assumed to be **harmonic** and generally are **anisotropic** (described by a symmetric second-rank tensor, or **six Bij parameters**).

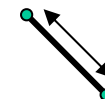
If a simple **isotropic model** is assumed, the magnitude of displacement is the same in all directions, the thermal ellipsoid becomes a sphere, and only **one parameter**, the **isotropic Displacement Parameter, Biso**, is necessary for atom k .

$$\mathbf{B}_k = 8\pi^2 \langle \mathbf{u}_k^2 \rangle$$

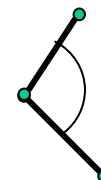
"A crystal should not be considered a chemical cemetery" (J. Dunitz)

Stereochemical restraints

Bond lengths, angles, etc. are parameters that must be reproduced by the model.



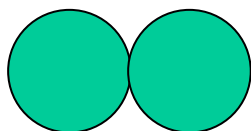
bond lengths



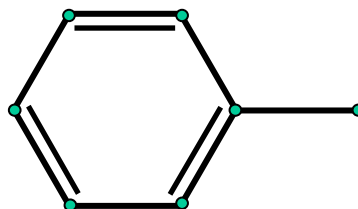
bond angles

$$S = \sum_{hkl} w[|F_{\text{obs}}| - |F_{\text{calc}}|]^2 + \sum_{\text{par}} (\sigma)^{-2}[d_{\text{obs}} - d_{\text{std}}]^2$$

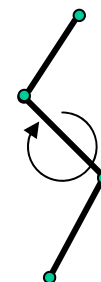
...



van der Waals contacts



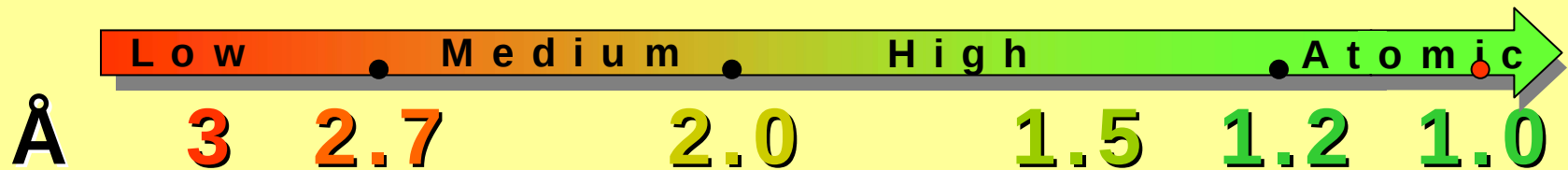
planar groups



torsion angles

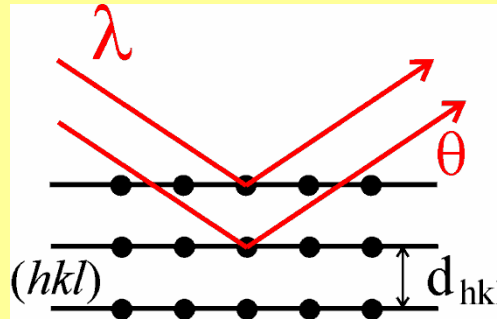
Resolution in crystallography

Resolution $d_{\min} = \lambda / (2 \sin \theta_{\max})$



Bragg equation:

$$\lambda = 2d \cdot \sin \theta$$



Sheldrick
criterion

it can be shown that the **technically defined** resolution $d_{\min}$ is almost exactly equivalent to **optical resolution**, i.e. the minimum separation of two points that are still distinguishable in the Fourier Transform of the diffraction data, i.e. in electron density map

In **small-molecule crystallography**, **atomic resolution** is nearly always achieved and **atomic coordinates** are determined directly as peak coordinates in Fourier or difference-Fourier maps

In **macromolecular crystallography** this is rarely the case; the **primary product** is a (lower resolution) **electron density map**, and an **atomic model** of a macromolecule is (at least to some degree) a result of its **subjective interpretation** by a crystallographer

small-molecule crystallography

(full Cu sphere, $\lambda=1.54 \text{ \AA}$)

$d_{\min} = 0.77 \text{ \AA}$

Mo radiation, $\lambda=0.71 \text{ \AA}$

$2\theta=60^\circ$

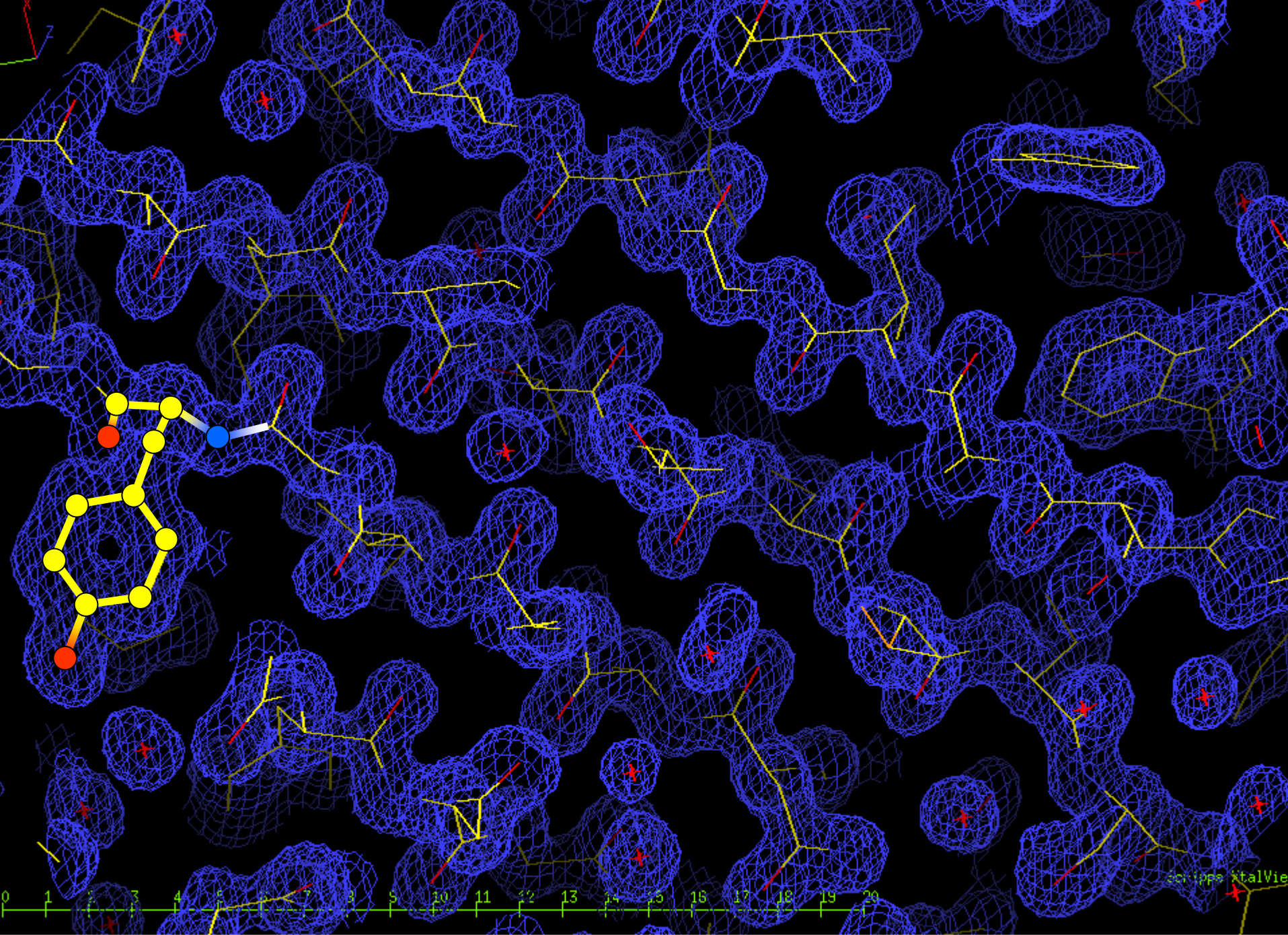
$d_{\min} = 0.71 \text{ \AA}$

macromolecular crystallography

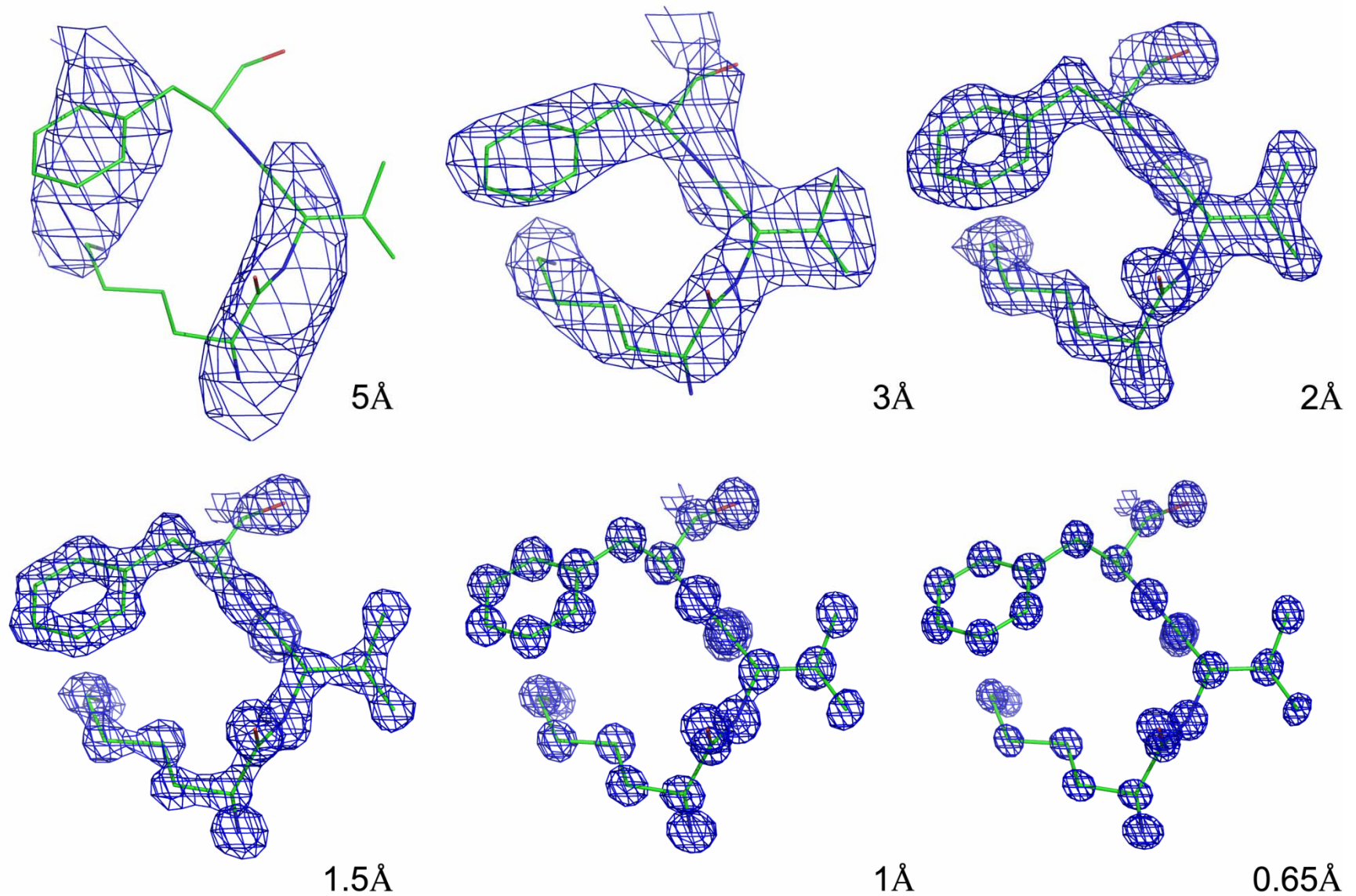
$d_{\min} = 1.2 \text{ \AA}$ (Sheldrick criterion)
(atomic resolution)

already a big success

but even if atomic resolution is not achieved –
atomic models can be built



Model building at low and high resolution



Resolution of electron-density maps and the corresponding model detail

Resolution	Features Observed
5.0 Å	Overall shape of the molecule
3.5 Å	C α trace
3.0 Å	Side chains
2.7 Å	Carbonyl O atoms (bulges) First chance to see water molecules
2.5 Å	Side chains well resolved, Peptide bond plane resolved
1.4 Å	Holes in aromatic rings Anisotropic B-factors possible
1.2 Å	True atomic resolution
1.0 Å	First chance to see H atoms
0.6 Å	Current limit for best protein crystals



Research Collaboratory for Structural Bioinformatics

1971 - Protein Data Bank created in Brookhaven Natl Lab (USA)
1998 - moved to RCSB (USA)
2003 - International organization (wwPDB)
Current Director – Dr. Helen Berman

<http://www.rcsb.org/pdb/>



The Worldwide Protein Data Bank (**wwPDB**)

is an international organization.

It consists of organizations that act as:

deposition, data processing and distribution centers for PDB data.

The founding members are:

[RCSB PDB](#) (USA), [PDBe](#) (Europe) and [PDBj](#) (Japan).

The [BMRB](#) (USA) group joined the wwPDB in 2006.

The mission of the wwPDB is to

maintain a single Protein Data Bank Archive

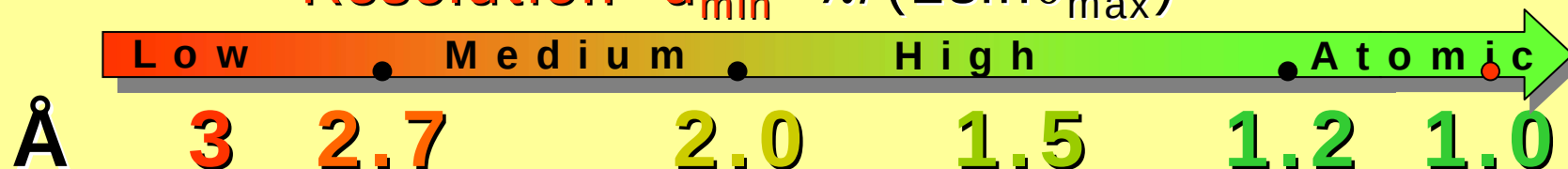
of macromolecular structural data

that is **freely and **publicly** available**

to the global community

How to assess crystallographic structures?

Resolution $d_{\min} = \lambda / (2 \sin \theta_{\max})$



R (residual)

R_{free} : how well does the model predict data it has never 'seen'?

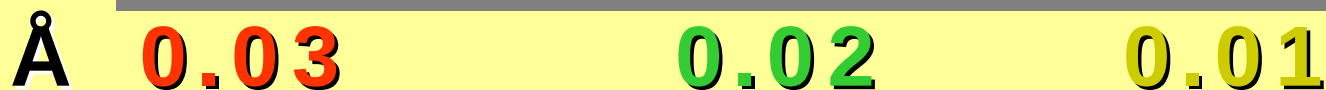
$$R = \frac{||F_o| - |F_c||}{\Sigma |F_o|}$$



$R_{\text{free}} - R$

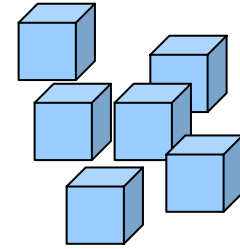


Deviations from „ideal” geometry
 $\text{rms}(\text{bond-length} - \text{standard})$

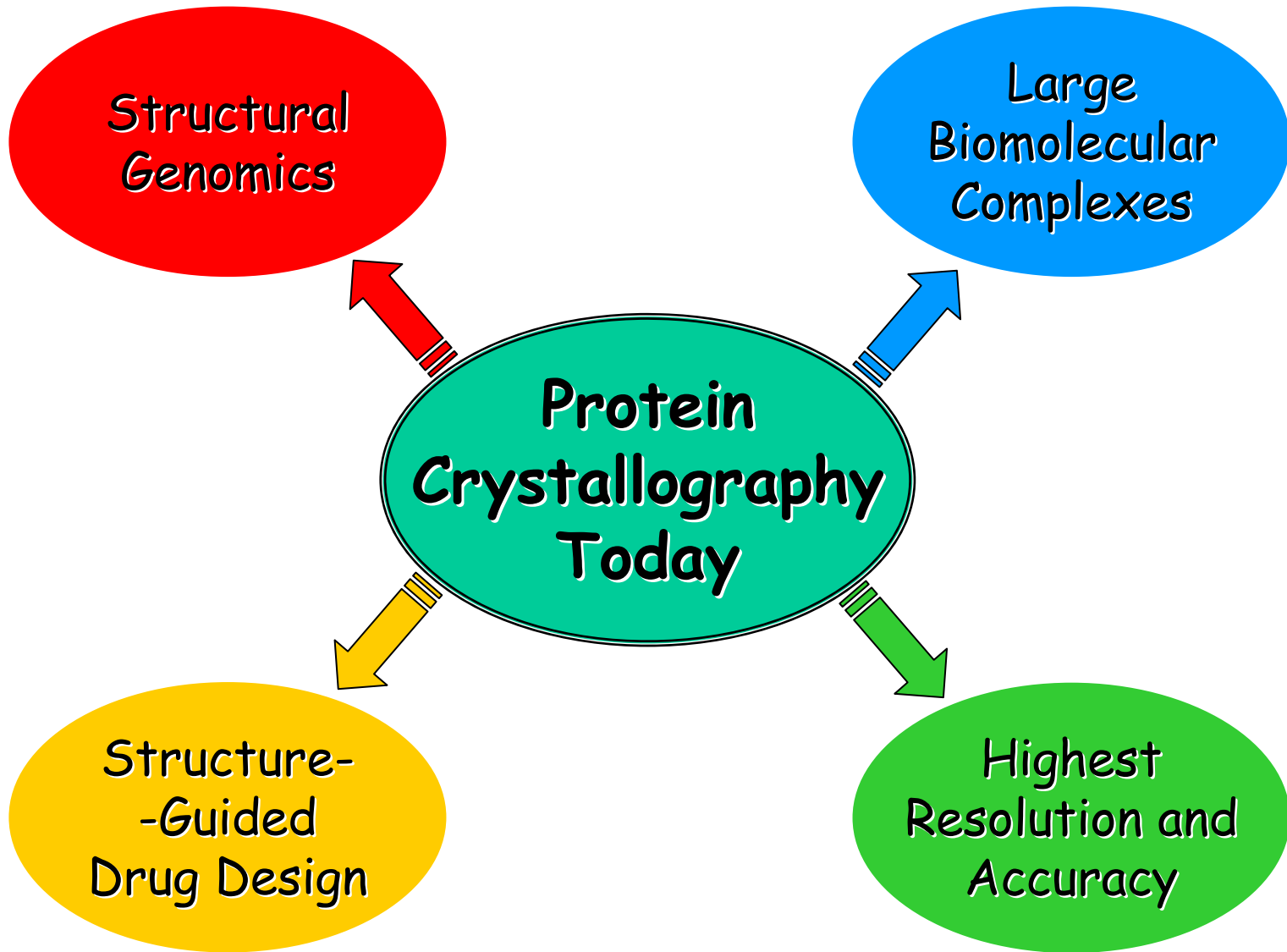


Protein crystallography

areas of advancement



- **advancements in crystallization methods**
- **development of cryocrystallographic techniques**
- **powerful synchrotron X-ray sources**
- **faster and more sensitive detectors**
- **faster computers**
- **better algorithms**



Structural Genomics

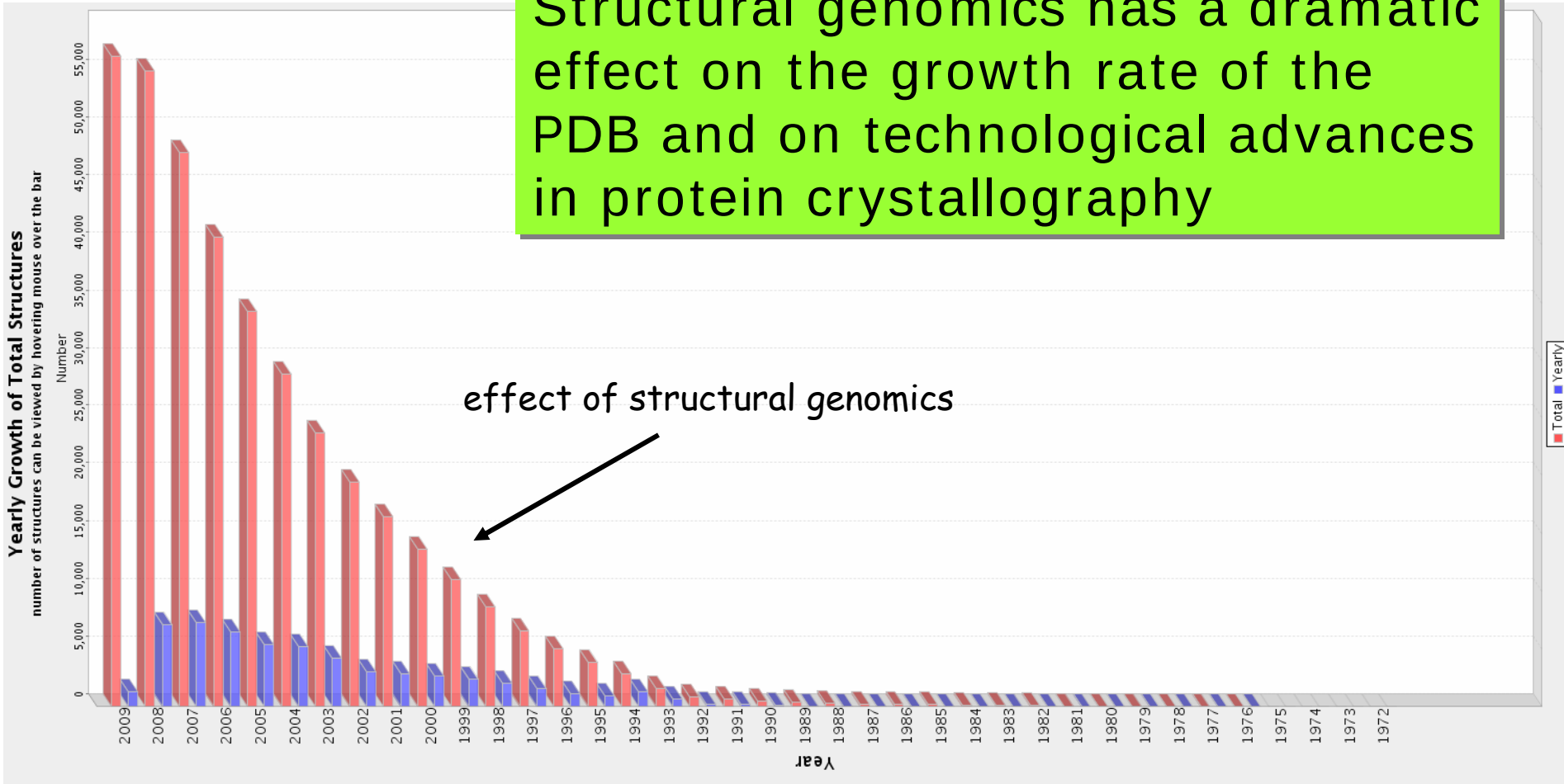
The aim of **structural genomics** is to determine in a **high-throughput** automated approach the 3D structure of **all the proteins** encoded in the genome of a given organism in order to understand their function. Contrary to classical biochemistry, the structure is studied before biochemical characterization of the target.

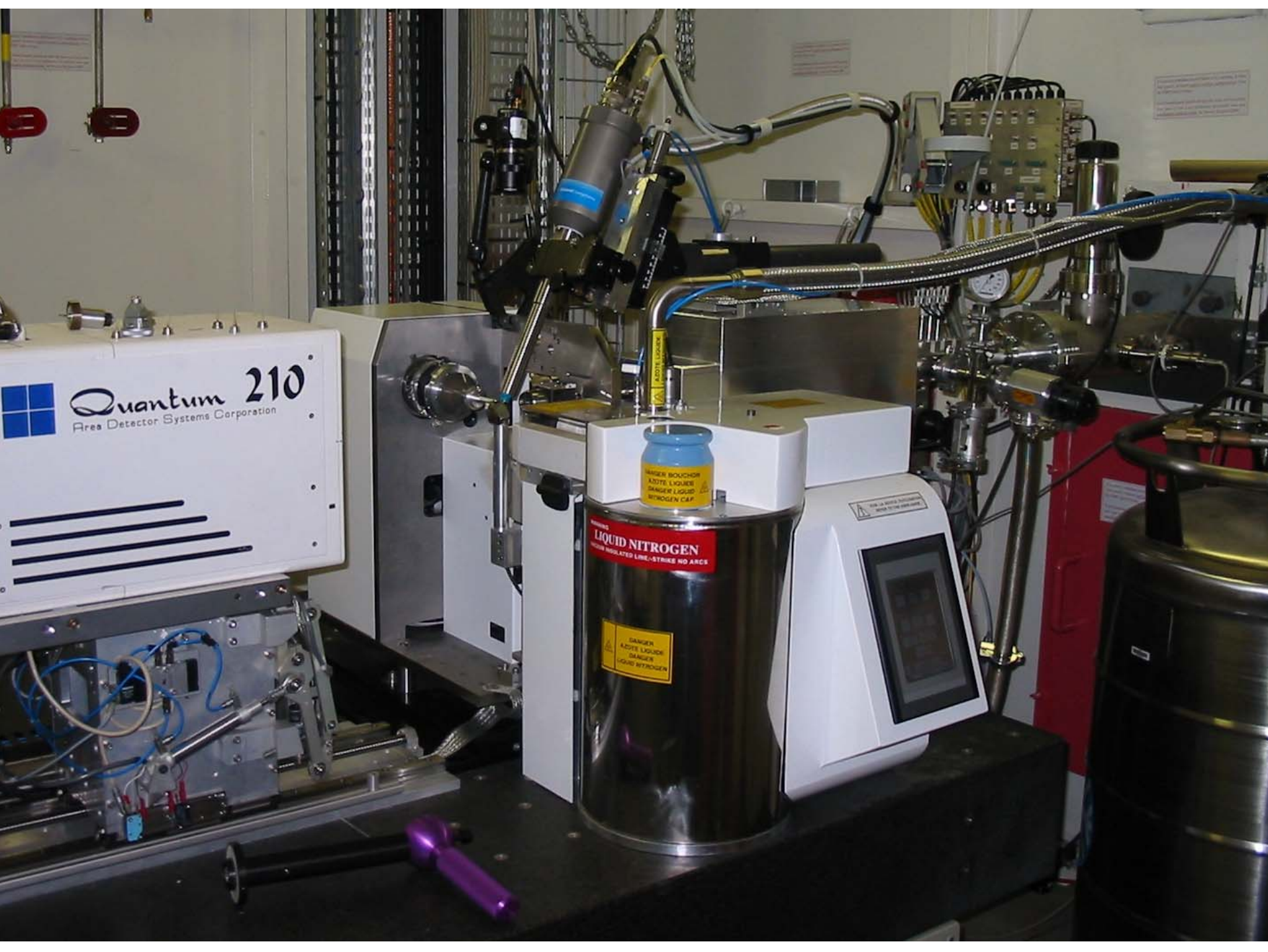
A number of human pathogens are being studied in this way, in search of new therapeutic targets.

Structural
Genomics

Growth of the PDB

Structural genomics has a dramatic effect on the growth rate of the PDB and on technological advances in protein crystallography

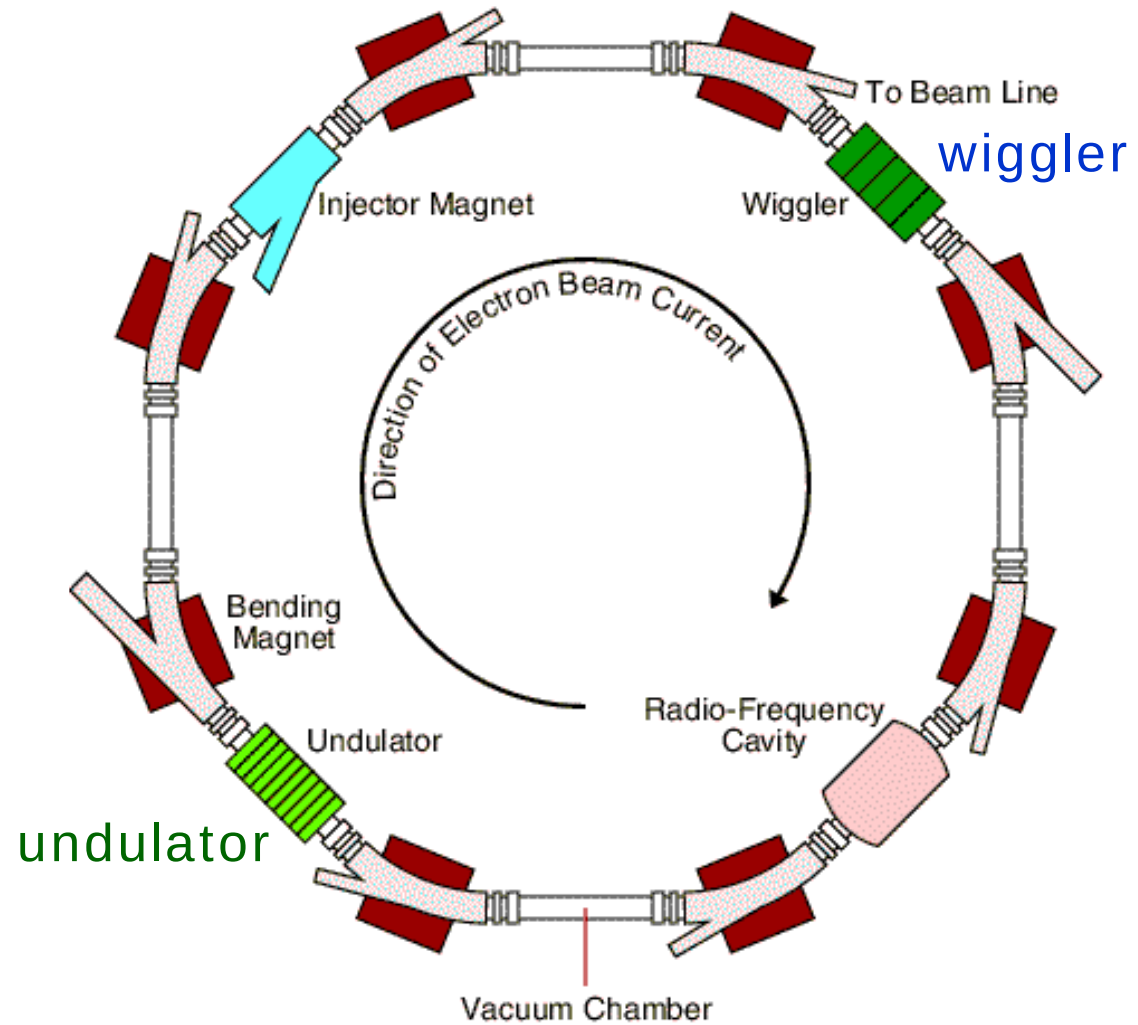
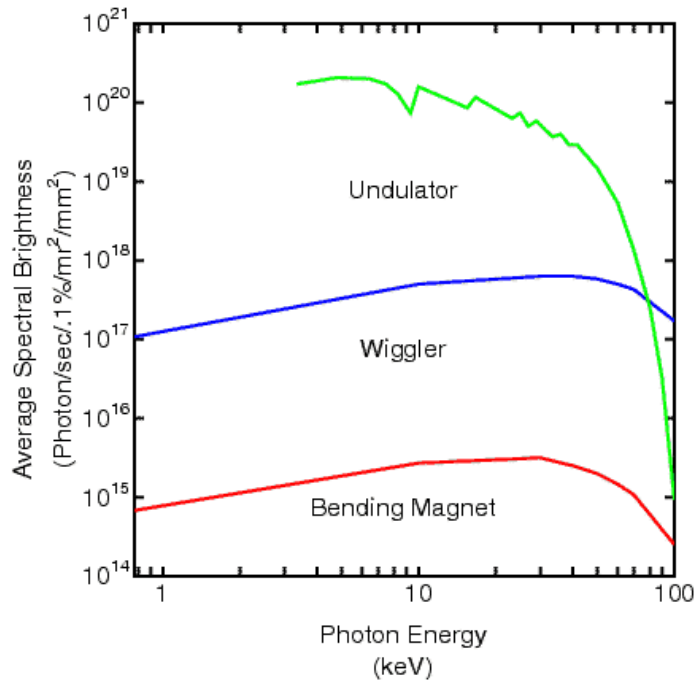
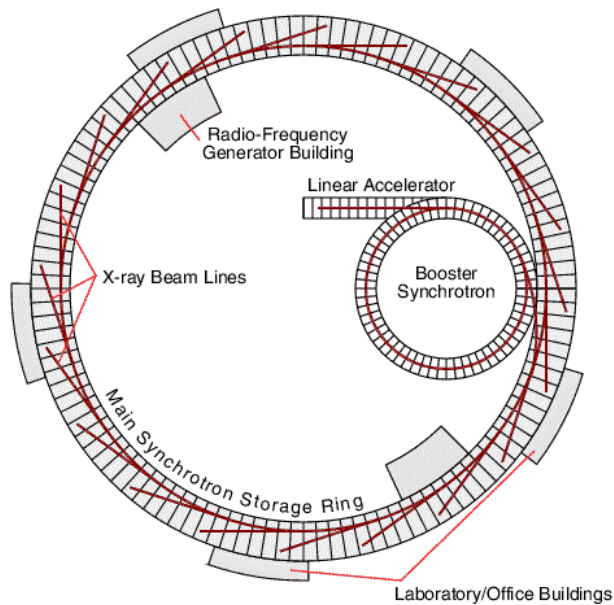




European Synchrotron Radiation Facility, Grenoble



Synchrotron

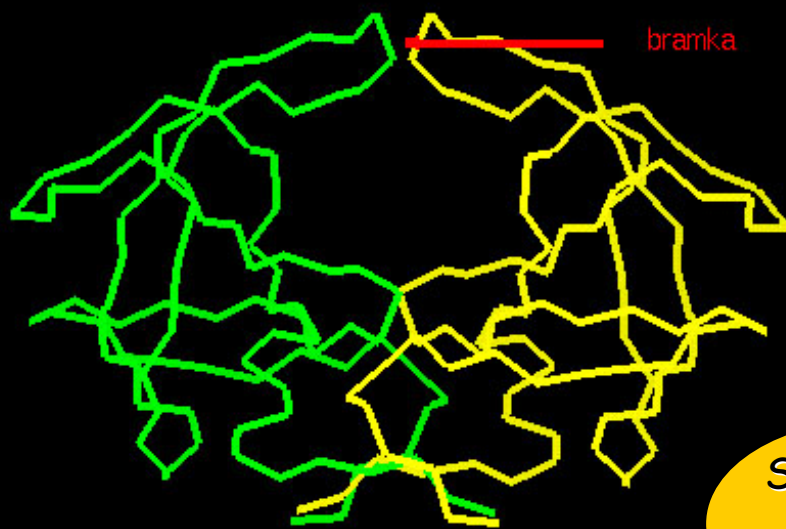


Characteristics of synchrotron radiation

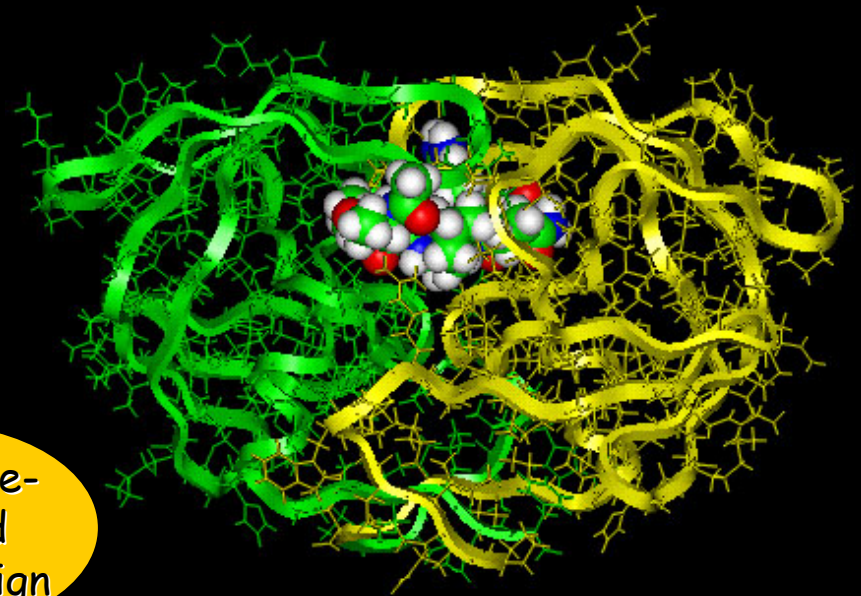
- Wide spectral range (λ)
- Tuneability
- Very high intensity
- Polarization (E in equatorial plane)
- Has a time structure

Poznan-ESRF remote access session

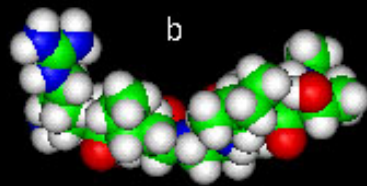
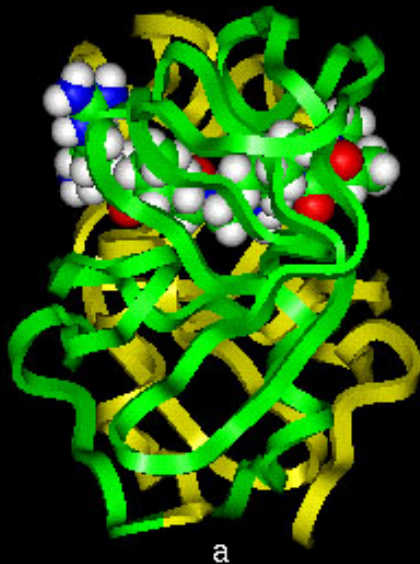




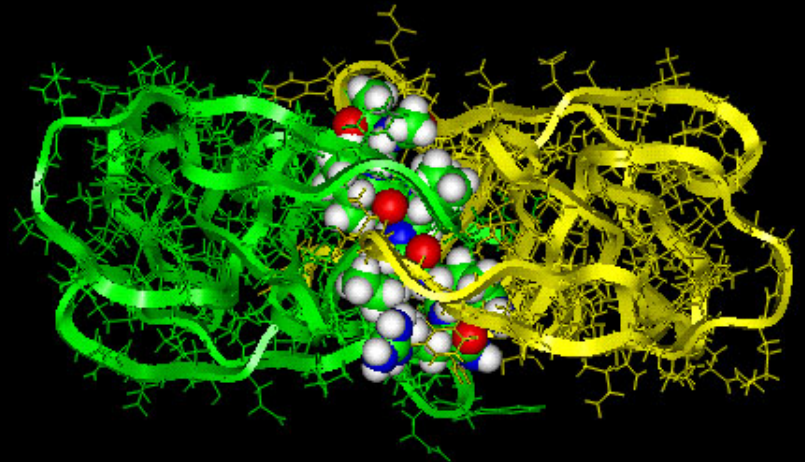
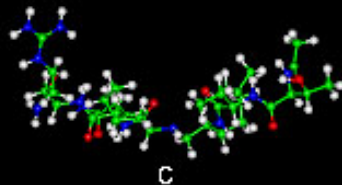
Structure-
-Guided
Drug Design



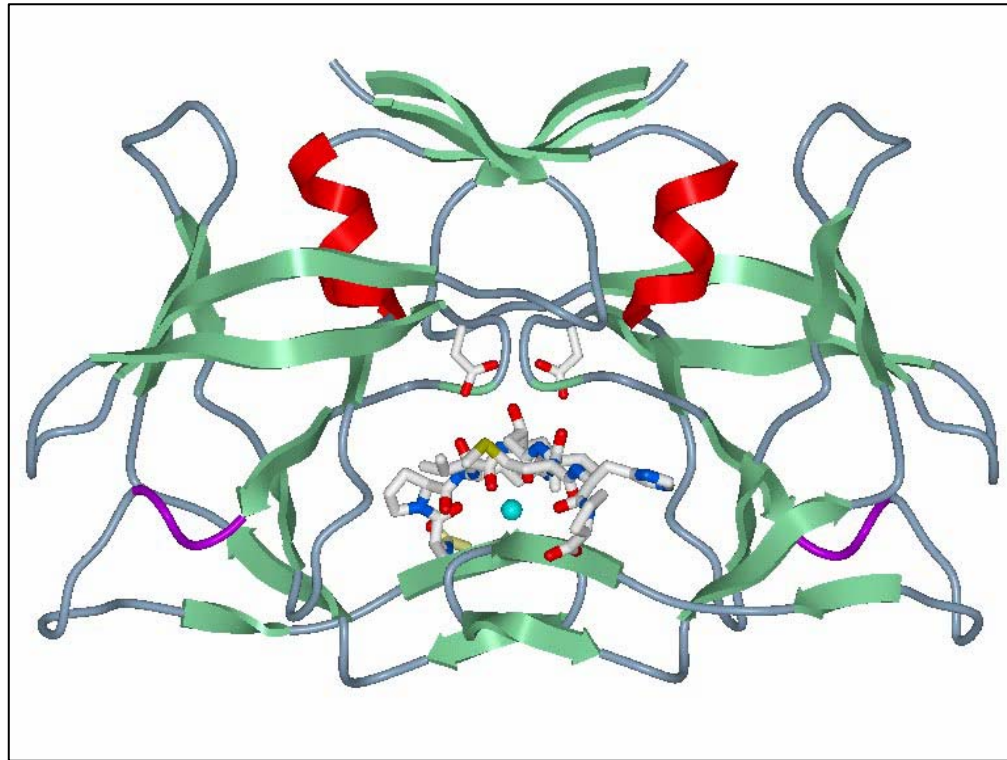
HIV-1 Protease

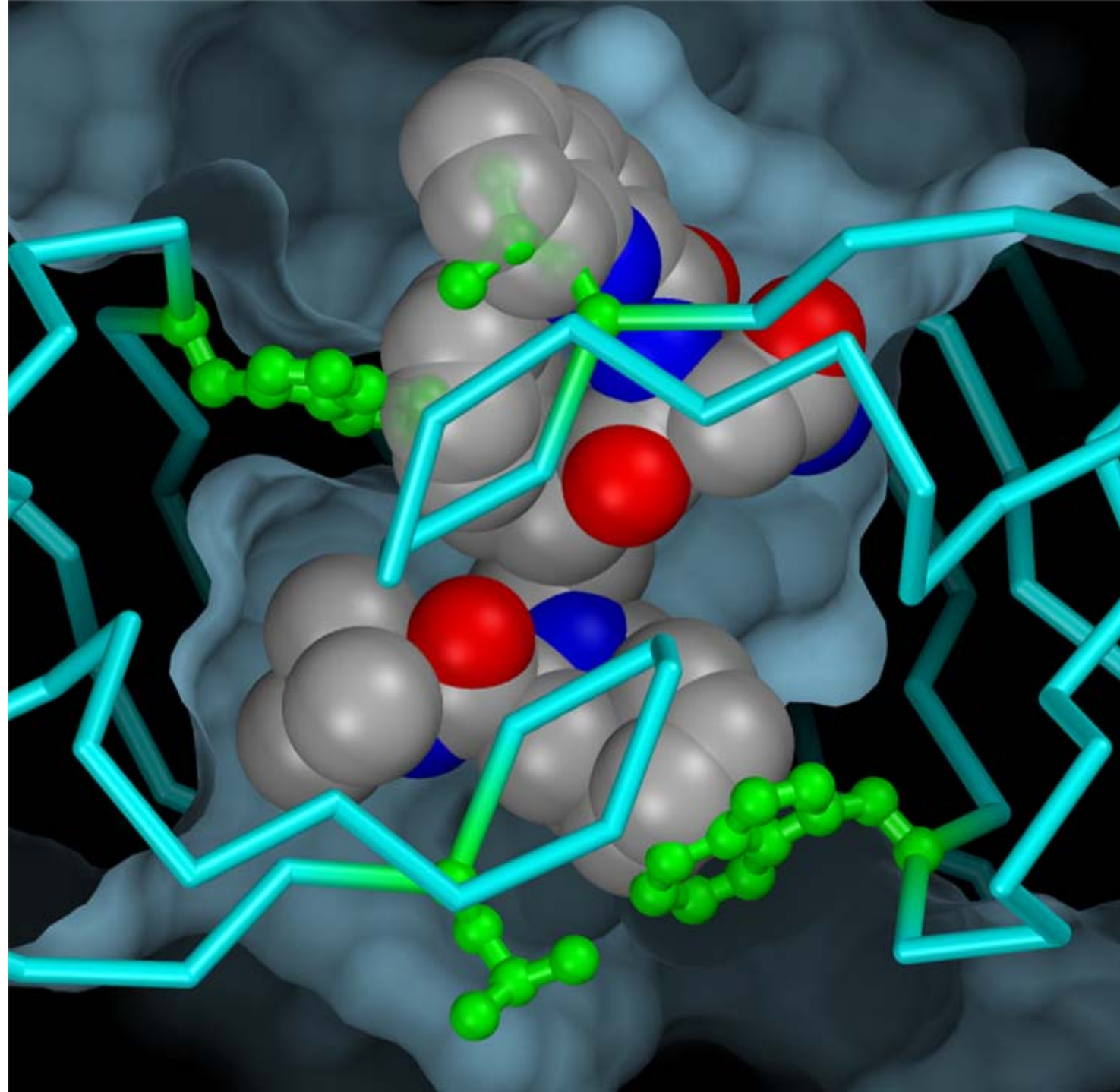


INHIBITOR
MVT - 101



Overall structure of HTLV-1 PR





Large
Biomolecular
Complexes

Examples:

- viruses
- the ribosome

Virus structure by X-ray crystallography

helical viruses

TMV

- Aaron Klug (1978) single-crystal X-ray diffraction, 2.8 Å
- Kenneth Holmes, Gerald Stubbs (1986) fiber X-ray diffraction, 3.6 Å

icosahedral (spherical) viruses

TBSV

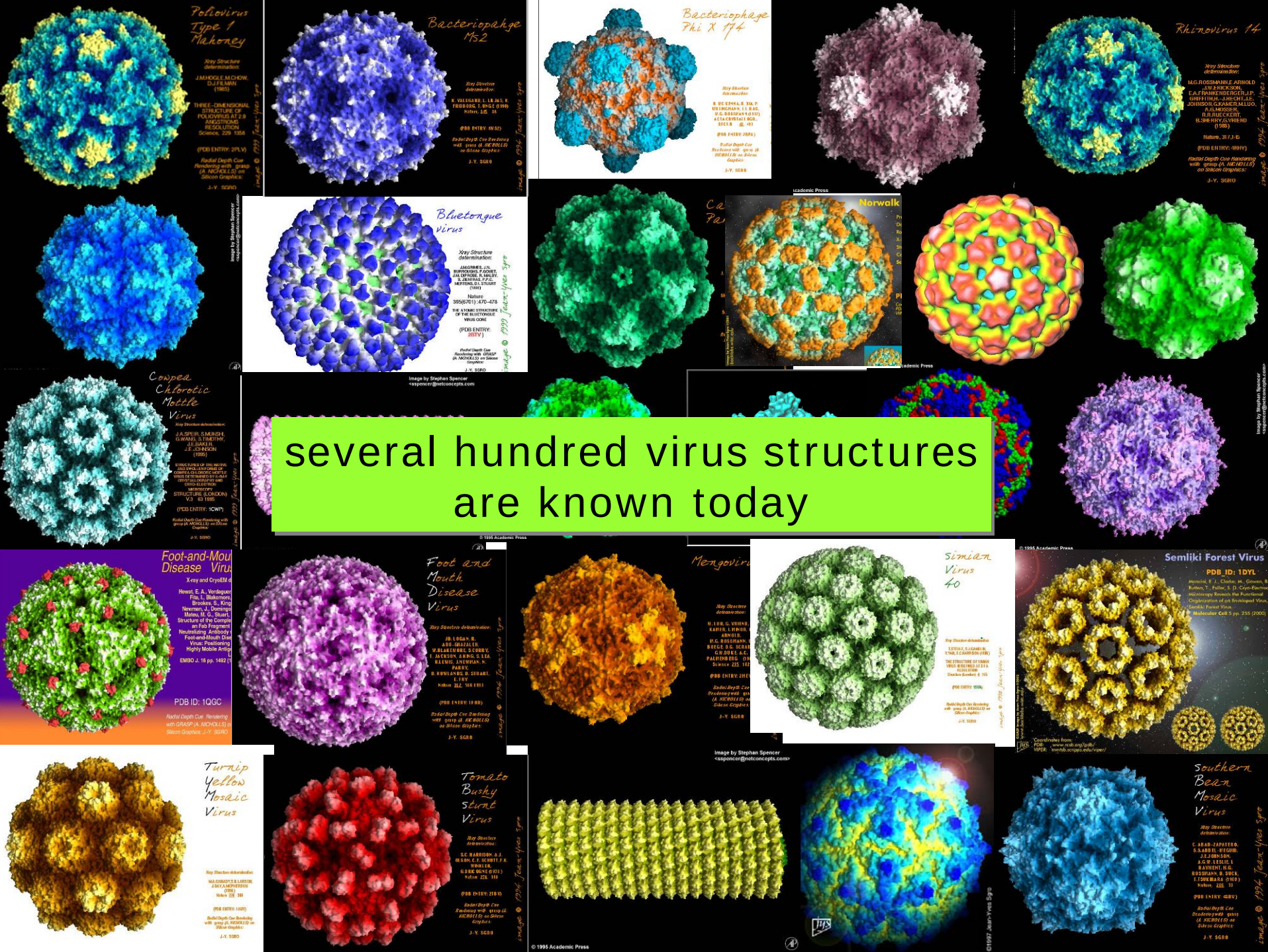
Bean mosaic virus

STNV

Polio virus

Common cold virus

- Stephen Harrison (1978) 2.9 Å
- Michael Rossmann (1980) 2.8 Å
- Lars Liljas (1984) 2.5 Å
- James Hogle (1985) 2.9 Å
- Michael Rossmann (1985) 2.8 Å



Ribosome – the protein factory of the cell

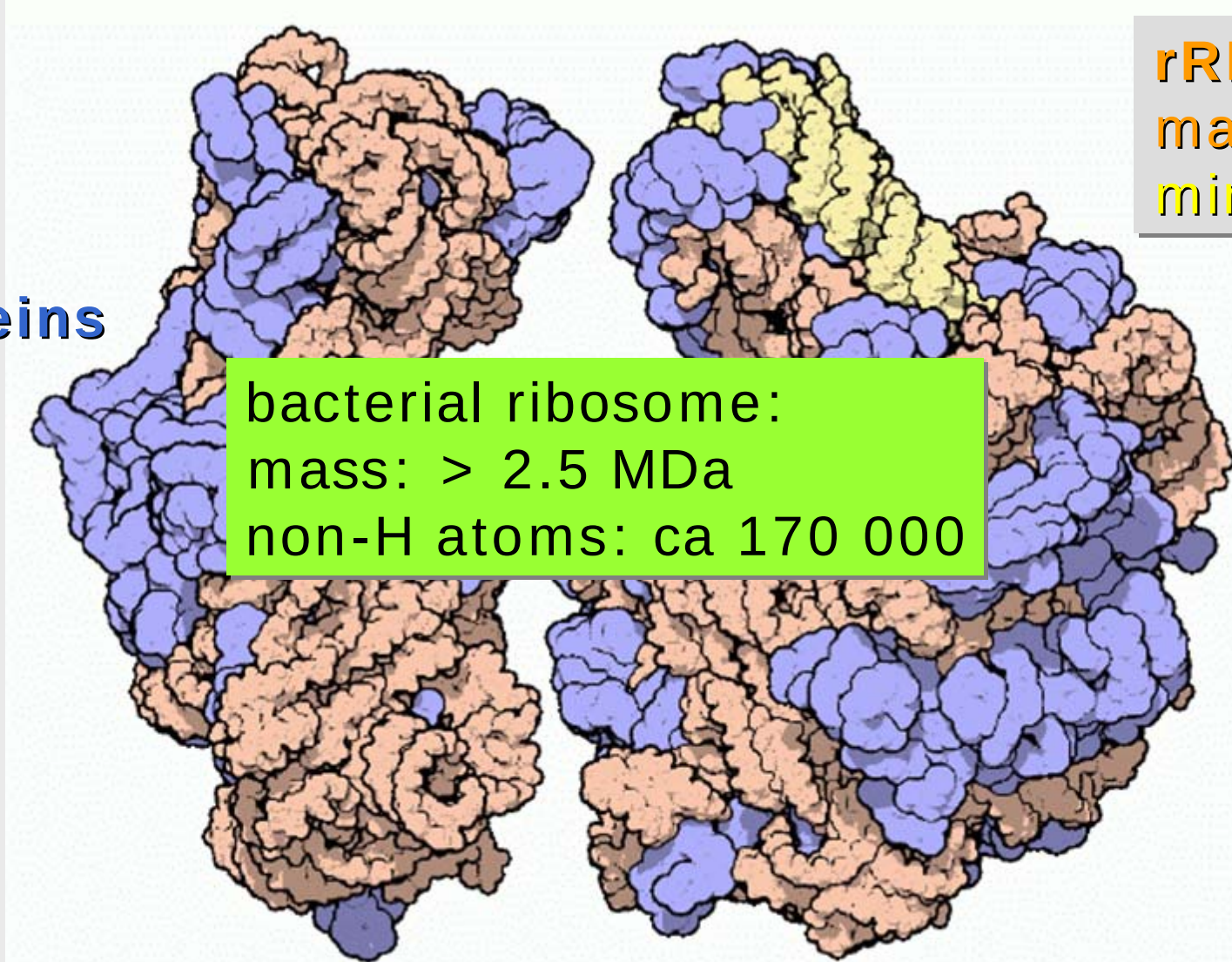
proteins

rRNA
major
minor

bacterial ribosome:
mass: > 2.5 MDa
non-H atoms: ca 170 000

small subunit

large subunit



Ribosome structure

- *small subunit, 3.0 Å, Venki Ramakrishnan et al., 2000; 3.3 Å, Ada Yonath et al., 2000*
- *large subunit, 2.4 Å, Thomas Steitz et al., 2000; 3.1 Å, Ada Yonath et al., 2003*
- ***complete ribosome with tRNA & mRNA, 5.5 Å, Harry Noller et al., 2001; 3.2 Å, Jamie Cate et al., 2006; 2.8 Å, Venki Ramakrishnan et al., 2006***

Ribosome structure

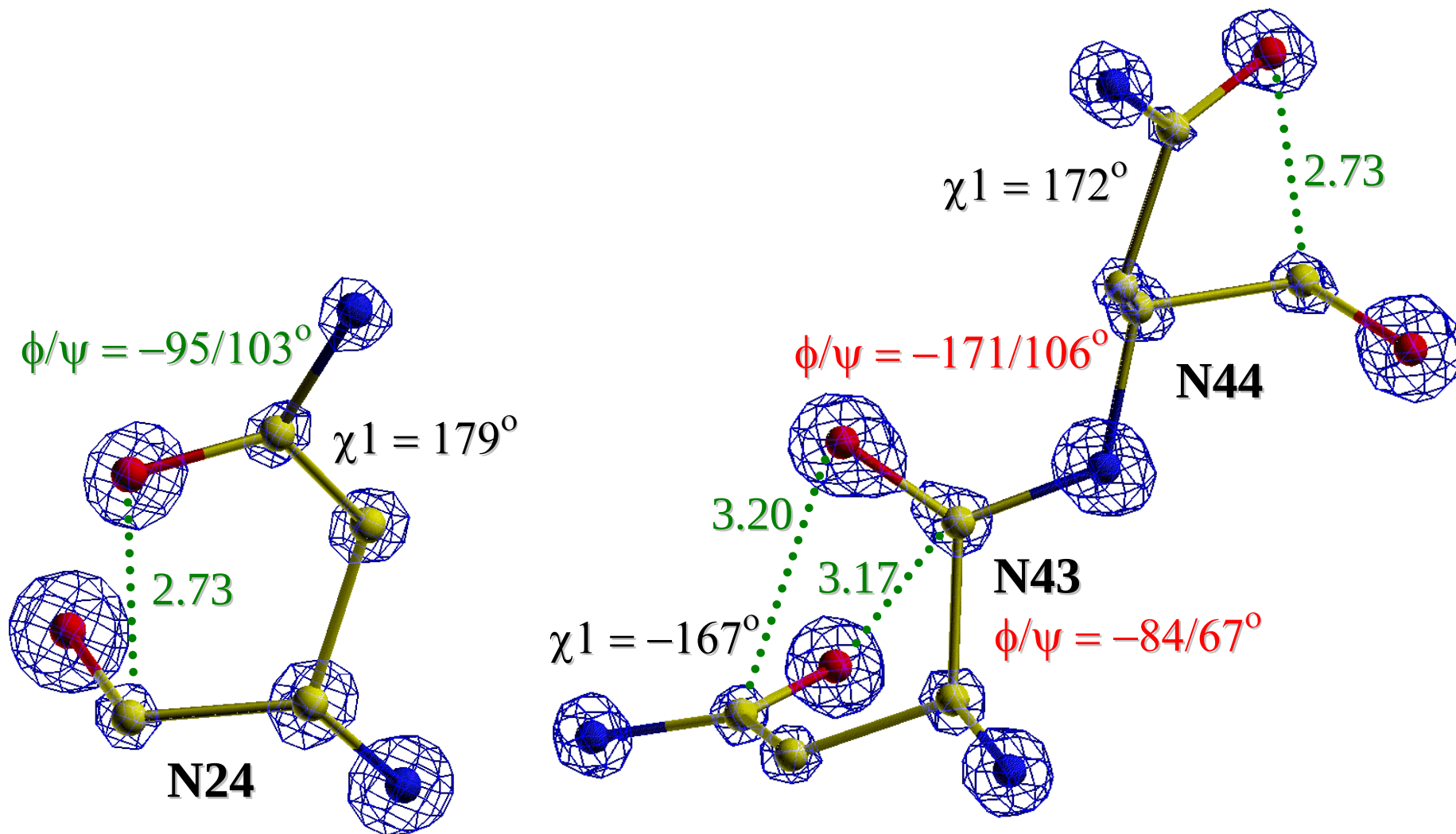


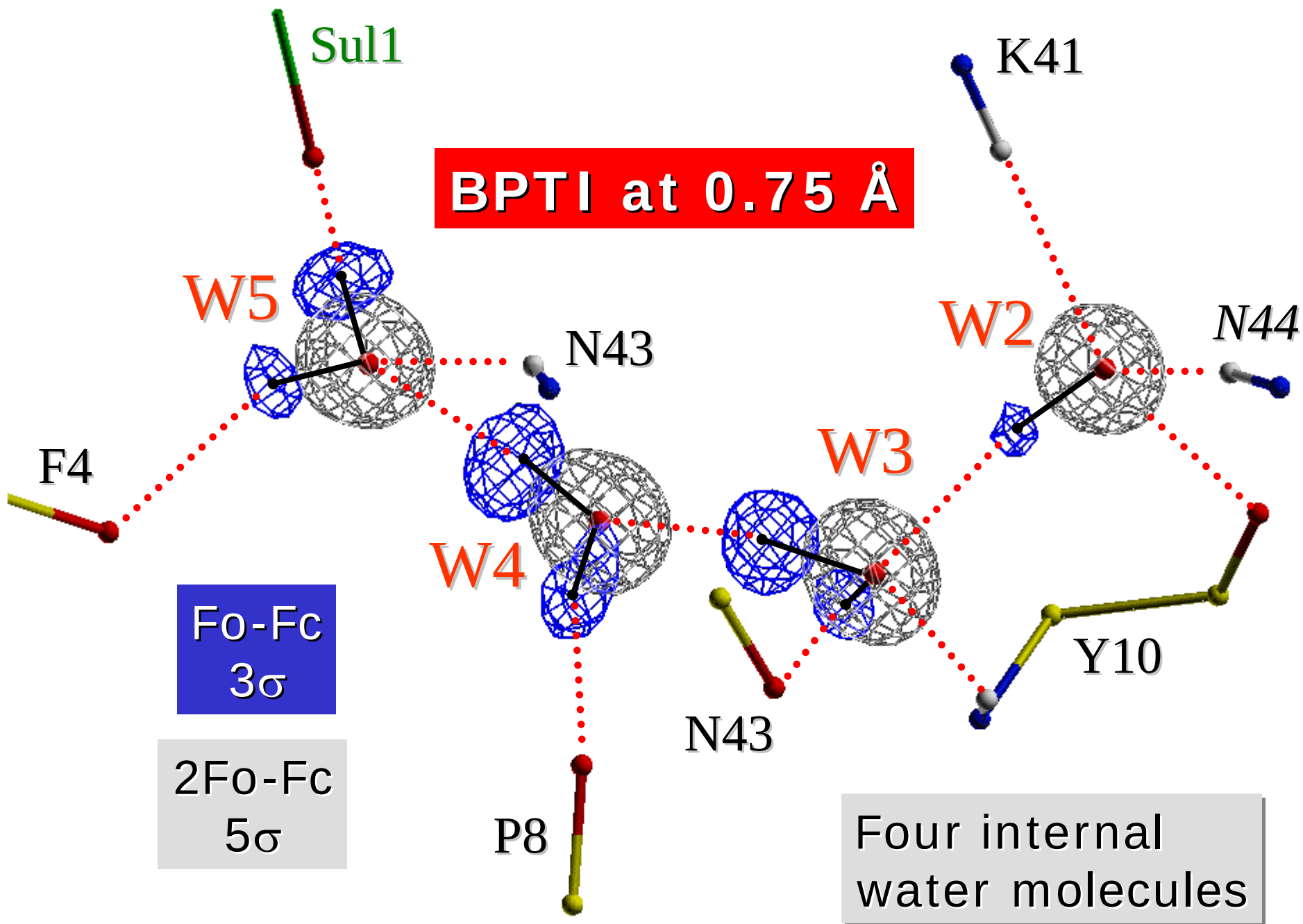
Highest
Resolution and
Accuracy

Record breaker: crambin at 0.54 Å

Examples from the structure of BPTI (Bovine
Pancreatic Trypsin Inhibitor) at 0.86 and 0.75 Å

BPTI at 0.86 Å resolution

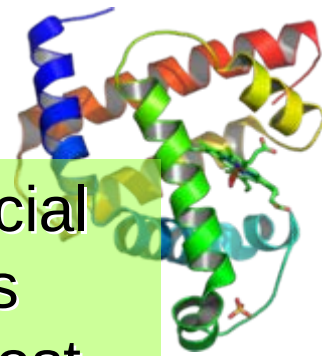




Protein Data Bank – growth of atomic-resolution entries

Resol (Å) at least	All holdings	1.4	1.2	1.0	0.8
1981	98	7	0	0	0
1986	224	13	3	1	0
1991	1103	40	15	7	0
1996	5929	116	44	16	0
2001	17694	590	284	102	6
July 09	59000	2800	950	360	20

5PTI - BPTI, Wlodawer & Huber (1984): N- and X-ray refinement



Crystallography – through the study of this special solid state of matter – crystalline protein, was historically the first method to reveal to us, almost exactly 50 years ago, the secret of protein structure. This was done for myoglobin by John Kendrew and for hemoglobin by Max Perutz.



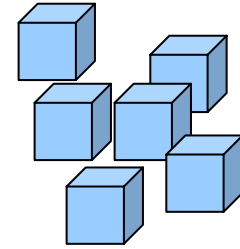
Today, we have accumulated in the PDB an enormous amount of structural information about proteins.

Crystallography is now assisted in this effort by other methods: NMR, electron microscopy, bioinformatics.

But crystallography still remains the main source of information about the structure of proteins, especially in the context of Structural Genomics.

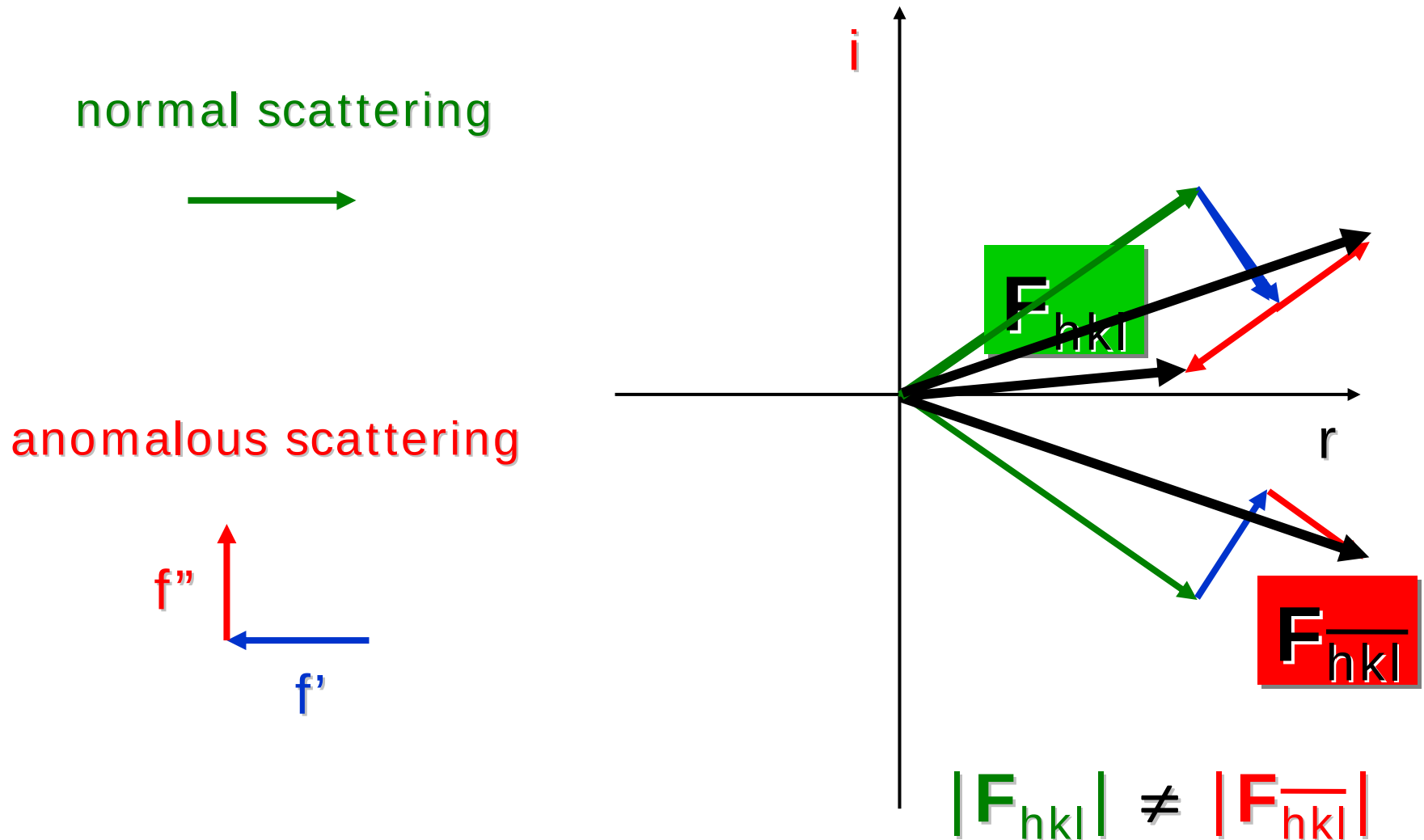
Protein crystals

high resolution – high goal



- only proper symmetry possible
- huge unit cells (ca 100 Å) – very weak diffraction ($I \sim V^{-2}$)
- huge numbers of reflections (e.g. 10^6 collected, 10^5 unique)
- gigantic molecules, often multiple, > 10,000 non-H atoms not rare
- molecules with a degree of flexibility - disorder
- high solvent content in crystal volume (50% or more)
- disorder in bulk water region
- crystals often sensitive and unstable

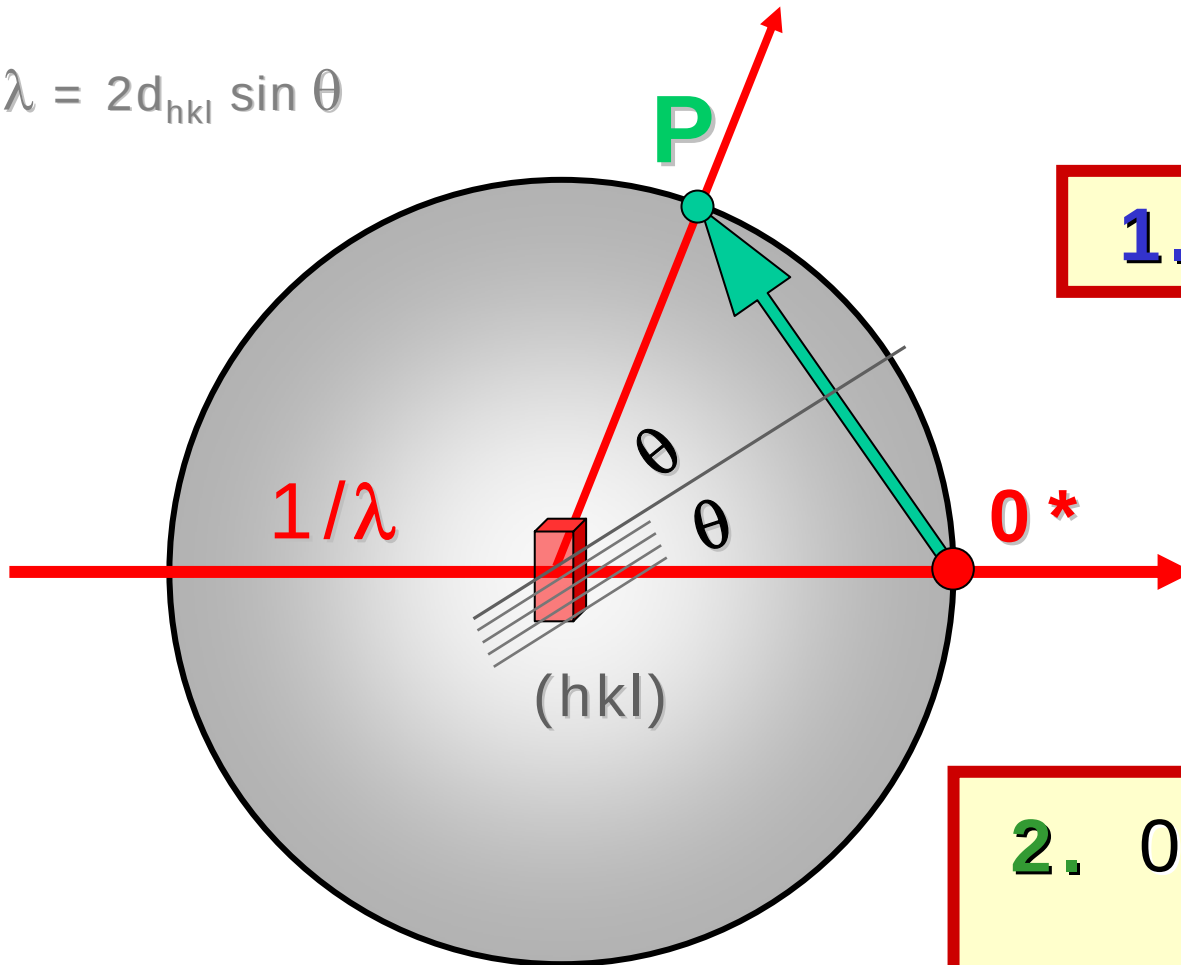
The Structure Factor in the case of anomalous scattering



The Ewald construction



$$\lambda = 2d_{hkl} \sin \theta$$



$$1. \quad 0^*P \perp (hkl)$$

$$2. \quad 0^*P = (2/\lambda)\sin\theta \\ = 1/d_{hkl}$$